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Non-linear effects on the resonant frequencies of a cantilevered plate

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ABSTRACT

In this paper, we address experimentally and theoretically the non-linear effects on the resonance of a periodically-forced cantilevered plate immersed in a fluid at rest. Experiments are performed with small aspect-ratio plates made of two different materials. When forced harmonically at their leading edges, these plates exhibit resonances for their first 3 structural modes. The frequencies at these resonances decrease when the forcing amplitude is increased, revealing the presence of non-linear effects. To model this phenomenon, a theoretical model is employed, which takes into account both resistive and reactive forces exerted by the fluid on the plate. By carrying out a weakly non-linear analysis, the frequencies at the resonances can then be determined. Model and experiments are in good agreement, showing that a weakly non-linear approach is suited to this kind of fluid–structure interaction and could be applied, in the future, to engineering problems such as energy harvesting with a fluttering plate or the biological problem of aquatic propulsion with a flexible fin.

1. Introduction

One of the most simple examples of fluid–structure interaction is the vibration of a cantilever plate in a fluid at rest. Yet, even in this simple example, complex effects may arise and hinder the development of accurate modeling. First, it should be realized that this fluid–structure interaction is strong in the sense that both the plate motion modifies the fluid flow around it and the flow modifies the forces on the plate and thus its dynamics. If structural resonances can be generated in the plate, fluid resonances can also occur, as well as fluid–structure ones (Crighton and Oswell, 1991; Shelley and Zhang, 2011). Second, as soon as the beam deflection is of the order of its length, non-linear effects come into play. These non-linear effects are mainly caused by geometrical effects (as the boundary conditions need to be evaluated on a displaced beam), but may also be caused by non-linearities of the structural constitutive relation (e.g. plastic deformation) or the fluid (e.g. if vorticity is detached). The objective of this paper is to study in an idealized system the weakly non-linear effects on the resonance of a cantilever plate.

Every structure has resonant frequencies associated to its elastic and geometric characteristics. In some cases, the resonant modes are excited on purpose, as for the two-stroke engine (Tenney, 1972; Blair, 1996), the string and wind instruments (Fletcher and Rossing, 1998), radio antennas (Huang and Boyle, 2008), magnetic resonance imaging (Kuperman,

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2000) and atomic force microscopes (Sader, 1998; Blom et al., 1992; Ghatkesar et al., 2008). These resonances may also be used to determine Young's modulus of a sample as it is done in a classical undergraduate experiment (Wilson and Lord, 1973; Turvey, 1990). In other cases, resonance is undesirable and may cause severe damages in architectural constructions, rail transport, aeronautics, or in the automobile industry. In this case, the excitation may be due to instabilities of the fluid–structure interactions such as flutter, galloping, or vortex-induced vibration (Dowell and Hall, 2001; Païdoussis, 1998, 2004).

When a plate vibrates, the forces that the fluid exerts on it can be decomposed into two components: reactive forces and resistive forces. Reactive forces have an inviscid origin and have been modeled in the context of fish swimming by Lighthill (1960, 1971) and recently revived by Candelier et al. (2011, 2013). Their origin is as follows: when a body immersed in a fluid moves, a certain added-mass of fluid is also displaced; the acceleration of this added-mass implies that the body has exerted a certain force on the fluid and that, reactively, the fluid has exerted the opposite force on the body. In the linear regime, or equivalently for small-amplitude deformations, added-mass effects tend to decrease the resonance frequencies of a structure compared to vibrations *in vacuo*, as they are equivalent to an increase in the structure mass (the resonant frequencies being inversely proportional to the square root of this mass).

Resistive forces are due to viscous effects and, in particular, are present when boundary layers separate and vorticity is detached. They result in drag forces that are, in the limit of large Reynolds numbers, proportional to the square of the normal velocity (Taylor, 1952). Resistive forces, because of this quadratic dependence, are inherently non-linear. They can be taken into account in a weakly non-linear model if some approximations are made (e.g. Lopes et al., 2002). It is usually assumed that resistive and reactive forces can be considered separately and that the total force exerted by the fluid is a linear superimposition of the two. Although this assumption could be disputed, we will proceed similarly in the present analysis.

In this work, we will consider both reactive and resistive forces acting on a vibrating plate. Our goal is to assess the importance of the first non-linear effects on the resonant frequencies. The theoretical model is adapted from a model developed by Eloy et al. (2012) and will be introduced in Section 2. Then, the setup and the experimental results will be presented in Section 3. Finally, a comparison between the two will be drawn in Section 4 before proposing a conclusion.

2. Model

2.1. Equation of motion

Consider a flexible rectangular plate of length L and width H clamped at one edge and free at the other (Fig. 1). When the clamped edge is set in motion in the y-direction with angular frequency ω, the deformation of the plate is one-dimensional, i.e. independent of the z-direction. In this case, its deflection can be described by the position vector $\mathbf{x}(s,t) = (x, y)$, which is a function of time and the curvilinear coordinate s measuring the distance from the clamped edge. This plate obeys the Euler–Bernoulli beam equation

$$m\partial_t^2\mathbf{x} + D\partial_s^4\mathbf{x} - \partial_s(\langle T\rangle\partial_s\mathbf{x}) + \langle p\rangle\hat{\mathbf{n}} = 0, \tag{1}$$

with m being the plate mass per unit area, D being the plate bending rigidity, and T being the generalized tension in the plate, which originates from the inextensibility condition. The last term in Eq. (1) comes from pressure forces because of a pressure jump $p(s,z,t)$ across the plate due to the surrounding flow. Finally, the $\hat{\mathbf{n}}$ denotes the unit vector normal to the plate and the brackets denote averaging in the z-direction along the plate width (Eloy et al., 2012). The equation of motion (1) is supplemented by the clamped-free boundary conditions: $y = \partial_s y = 0$ in $s=0$, and $\partial_s^2 y = \partial_s^3 y = T = 0$ in $s=L$.

In the elongated body limit (i.e. $H \ll L$), pressure forces on the plate can be decomposed into two parts

$$\langle p\rangle = p_{\text{reac.}} + p_{\text{resis.}}, \tag{2}$$

with $p_{\text{reac.}}$ being the reactive force and $p_{\text{resis.}}$ being the resistive force. The reactive force is physically due to the necessity for the plate to accelerate a certain added mass of fluid when it is set in motion. To accelerate this mass of fluid, a force has to be exerted on it, and reactively the opposite force is exerted on the plate (Lighthill, 1971). The resistive force is simply the drag on the plate due to the crossflow component of the relative velocity between the plate and the fluid. These forces can be written as

$$p_{\text{reac.}} = M\left(\dot{w} - (uw)' + \tfrac{1}{2}w^2\kappa\right), \tag{3}$$

$$p_{\text{resis.}} = \frac{1}{2}\rho C_d |w| w, \tag{4}$$

where κ is the plate curvature, $M = \pi\rho H/4$ is the added mass of air per unit area, C_d is a drag coefficient taken to be $C_d = 1.8$ for a plate (Buchak et al., 2010), dots and primes denote differentiation with respect to t and s respectively, u and w are the longitudinal and normal components of the plate velocity respectively such that $\dot{\mathbf{x}} = u\hat{\mathbf{t}} + w\hat{\mathbf{n}}$ and

$$u = \dot{x}x' + \dot{y}y', \quad w = -\dot{x}y' + \dot{y}x'. \tag{5}$$