Journal of the Mechanics and Physics of Solids 121 (2018) 281–295

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

# Fishnet model with order statistics for tail probability of failure of nacreous biomimetic materials with softening interlaminar links

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ARTICLE INFO

*Article history:*
Received 4 May 2018
Revised 30 June 2018
Accepted 29 July 2018
Available online 2 August 2018

*Keywords:*
Failure probability
Fracture mechanics
Structural strength
Monte carlo simulations
Lamellar structures
Material architecture
Structural safety
Size effect
Scaling
Probability distribution function (pdf)
Quasibrittle materials
Brittleness

ABSTRACT

The staggered (or imbricated) lamellar “brick-and-mortar” nanostructure of nacre endows nacre with strength and fracture toughness values exceeding by an order of magnitude those of the constituents, and inspires the advent of new robust biomimetic materials. While many deterministic studies clarified these advantageous features in the mean sense, a closed-form statistical model is indispensable for determining the tail probability of failure in the range of 1 in a million, which is what is demanded for most engineering applications. In the authors’ preceding study, the so-called ‘fishnet’ statistics, exemplified by a diagonally pulled fishnet, was conceived to describe the probability distribution. The fishnet links, representing interlaminar bonds, were considered to be elastic perfectly-brittle. However, the links may often be quasibrittle or almost ductile, exhibiting gradual post-peak softening in their stress-strain relation. This paper extends the fishnet statistics to links with post-peak softening slope of arbitrary steepness. Probabilistic analysis is enabled by assuming the postpeak softening of a link to occur as a series of finite drops of stress and stiffness. The maximum load of the structure is approximated by the strength of the $k$th weakest link ($k \geq 1$), and the distribution of structure strength is expressed as a weighted sum of the distributions of order statistics. The analytically obtained probabilities are compared and verified by histograms of strength data obtained by millions of Monte Carlo simulations for each of many nacreous bodies with different link softening steepness and with various overall shapes.



## 1. Introduction

The strength and fracture energy of nacre, the shell of pearl oyster or abalone, exceeds by an order-of-magnitude the strength of its constituents (95% $CaCO_3$). This remarkable property has been shown to originate from the imbricated (or staggered) ‘brick-and-mortar’ arrangement of nanoscale aragonite platelets bonded by a bio-polymer (Gao et al., 2003; Shao et al., 2012; Wang et al., 2001; Wei et al., 2015). Thus, the nacre’s nanostructure is of great interest for developing new ultra-strong and ultra-tough biomimetic materials.

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https://doi.org/10.1016/j.jmps.2018.07.023
0022-5096/

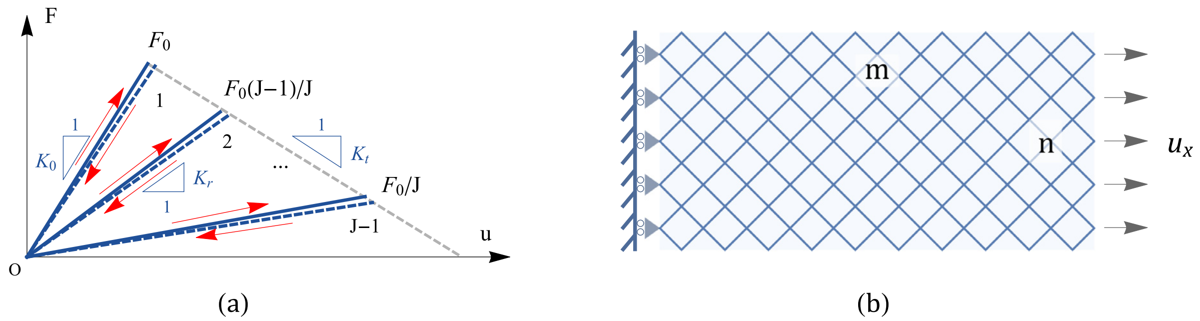


**Fig. 1.** (a) Numerical treatment of softening behavior of each single link; (b) Schematic showing model configuration and loading conditions.

Most studies have so far been deterministic. However, it is generally agreed that most engineering structures (bridges, airframes, electronic components, etc.) must be designed for failure probability not exceeding $10^{-6}$ per lifetime (which is negligible compared to the risk of death in car accident, $10^{-2}$, and is similar to the risk of death by a falling tree, by lightning, etc.). To design for failure risk $< 10^{-6}$, a theoretically based analytical closed-form probability distribution is indispensable. A direct experimental verification of the distribution, by histograms testing, is impossible because $> 10^8$ test repetitions would be required to verify the distribution tail at $10^{-6}$, which is obviously beyond reach. Therefore, the experimental verification must rely on other predictions, and the predicted size effect is most useful.

A previous study (Luo and Bažant, 2017a; 2017b) presented a new statistical model, the fishnet statistics, that can predict the probability tail of a nacreous material in which the fishnet links are perfectly brittle, i.e., their stress drops suddenly to zero as soon as the strength limit of the link is reached. The strength probability distribution of a fishnet was shown to lie between those of a fiber bundle (Daniels, 1945; Salviato and Bažant, 2014) and of a finite (or infinite) chain (Bažant, 2005; Bažant and Le, 2017; Bažant et al., 2009; Bažant and Pang, 2007; Bažant and Planas, 1998; Bažant and Pang, 2006; Le and Bažant, 2009; 2011; Le et al., 2011). The distances from the mean to the point of $10^{-6}$ failure probability differ, for these two limiting cases, by about 2:1, and the fishnet distribution can provide a continuous transition between these two limiting distributions.

In this study (posted in preliminary form as ArXiv (Luo and Bažant, 2018)), we extend the fishnet statistics to links that are quasibrittle, exhibiting progressive postpeak softening of various steepness. The softening may give a more realistic characterization of the interlaminar bond failures in some nacreous structures.

Before reaching the maximum load (which indicates the stability limit and failure if the load is controlled), the fishnet may already contain various numbers, 0, 1, 2,... , of failed links. Based on this observation, the fishnet statistical model splits the fishnet survival event into a union of disjoint events corresponding to different numbers of failed links, which implies a summation of survival probabilities:

$$1 - P_f(\sigma) = P_{S_0}(\sigma) + P_{S_1}(\sigma) + P_{S_2}(\sigma) + \cdots \tag{1}$$

where $P_f$ is the failure probability and $P_{S_k}(\sigma)$ ($k = 1, 2, 3, \ldots$) are the probabilities of the whole fishnet surviving under load (or nominal stress) $\sigma$ while there are exactly $k$ failed links under load $\sigma$. This formulation works quite well for fishnets with brittle links but it also poses two difficulties.

First, in obtaining the second and third term ($P_{S_1}$ and $P_{S_2}$) in the foregoing expansion, an equivalent uniform redistributed stress needs to be used for regions near the failed link, based on the stress field from finite element simulation. The error of doing so is negligible when we truncate the expression at the second or third terms (which was shown to give, for brittle links, sufficient accuracy). But it becomes considerable as more higher-order terms are added. This is because, at the lower tail, $P_{S_k}$ is of the same magnitude as $P_1(\sigma)^k$, and so the higher-order terms can be easily ruined by the errors from the previous terms.

Second, as the links become less brittle, more widely scattered damages tend to occur before the peak load, and so more higher-order terms need to be included to predict the failure probability accurately. It is, unfortunately, far more tedious to calculate them. In the previous study, $P_{S_2}$ had to be separated into two parts, to distinguish the cases of two failed links which are either close to, or far away, from each other.

Proceeding similarly, one would have to partition the higher-order terms based on the relative positions of the $k$ failed (or damaged) links and track the stress history for each single case. As $k$ increases, the formula would become too complicated. So the existing fishnet model is suitable only when the links are brittle or almost brittle, in which case it suffices to consider only a few terms in the fishnet expansion (Eq. (1)).

For fishnets with softening links, a modified approach is needed to calculate the failure probability. Although the brittle fishnet model is not applicable to a softening fishnet, its concept has inspired two key ideas to tackle the softening fishnet:

(1) Instead of a sudden drop of link stiffness to zero stiffness, the progressive continuous postpeak softening is decomposed into a series of sudden stiffness drops, each of them from one link stiffness to the next lower stiffness (Fig. 1a).