Journal of the Mechanics and Physics of Solids 000 (2018) 1–19

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Searching for imperfection insensitive externally pressurized near-spherical thin shells

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ARTICLE INFO

Article history:
Received 10 September 2017
Revised 24 May 2018
Accepted 6 June 2018
Available online xxx

Keywords:
Spherical shells
Buckling
Imperfection sensitivity

ABSTRACT

This paper studies the buckling behavior and imperfection sensitivity of geodesic and stellated shells subject to external pressure. It is shown that these structures can completely eliminate the severe imperfection sensitivity of spherical shells and can achieve buckling pressure and mass efficiency higher than the perfect sphere. Key results of this paper are as follows. First, a shell with the shape of an icosahedron can carry external pressure significantly higher than a spherical shell, when the effects of geometric imperfections are considered. Second, stellated shells are generally insensitive to imperfections. For pyramids with height-to-radius ratios greater than 35% the buckling pressure is greater than for a perfect sphere. The specific ratio 45% gives the highest buckling pressure, 28% higher than the perfect sphere. Third, stellated icosahedra with concave pyramids have higher mass efficiency than the perfect sphere. Fourth, in terms of volume efficiency, geodesic shells are comparable to spherical shells with a knockdown factor of 0.2 and convex stellated shells are comparable to spherical shells with a knockdown factor of 0.65.

1. Introduction

The sphere is the smallest area surface that encloses a given volume, and this characteristic has made spherical shells important structural components for load-carrying and space-confinement, and also common forms in nature. The hypothetical concept of flying ships hanging from ultralight spheres with internal vacuum, proposed in the1600s (Lana-Terzi, 1670) is still waiting for advances in structures and materials to become feasible. Recent studies by Palazotto and co-workers (Adorno-Rodriguez and Palazotto, 2015; Metlen, 2012; Snyder and Palazotto, 2017) of vacuum near-spherical structures have shown that a self-buoyant icosahedron consisting of a lightweight skin supported by a frame is now theoretically possible.

Current applications of large-scale spherical shells include gas containers in the petroleum industry, fuel tanks for rockets, deep-sea vehicles, and many others. At the micro/nano-meter scale, spherical shells have been recently used as colloidal capsules for drug delivery in biomedical engineering (Jose et al., 2014). Spherical and near-spherical shells in biological structures, such as viral capsids, have recently attracted significant attention from both academic and medical communities (Lidmar et al., 2003; Mannige and Brooks, 2009; May and Brooks, 2012; Ru, 2009).

Spherical shells subject to external pressure exhibit highly nonlinear postbuckling behavior with dramatic sensitivity to even very small imperfections (Hutchinson, 1967, 2016). In the 1930s this extreme imperfection sensitivity was identified,

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https://doi.org/10.1016/j.jmps.2018.06.008

and Von Kármán and Tsien were among the first to carry quantitative studies of the discrepancies between theory and experiments (Von Kármán and Tsien, 1939). In the1940s, Koiter proposed a general theory of elastic stability to calculate the buckling sensitivity to small imperfections (Koiter, 1945). Following Koiter's seminal work, theoretical and experimental investigations of the imperfection sensitivity of pressurized spherical shells reached a climax in the 1960s, although it should be noted that, compared to axially loaded cylindrical shells —which also show high imperfection sensitivity— the spherical shell research community remained smaller (Carlson et al., 1967; Hoff and Soong, 1963; Thompson, 1960). An extensive review of the literature on buckling of spherical shells has been provided by Hutchinson (2016).

Currently, there is renewed interest in this topic, motivated by recent advances in digitally-based design and shape optimization, as well as fabrication technologies that have enabled precisely engineered shapes (Jimenez et al., 2016; Lee et al., 2016a,b). In parallel, advances in the life sciences have drawn attention to medical implications of the buckling of spherical (or sphere-like) viral and colloidal capsules under external osmotic pressure (Datta et al., 2012; Kim et al., 2014; Vliegenthart and Gompper, 2011). Recent research has focused on differences in behavior caused by changing the boundary conditions from dead pressure to volume control (Thompson and Hutchinson, 2017).

From a practical perspective, current methods for the design of spherical shells under external pressure are still primitive. The empirical knockdown-factor method is the main method to account for the reduction in theoretically estimated buckling pressure due to imperfections. The buckling pressure is estimated from:

$$P_{cr} = \gamma P_{cl} \tag{1}$$

where γ is an empirically based knockdown factor and P_{cl} is the classical buckling pressure of a perfect shell (Zoelly, 1915):

$$P_{cl} = \frac{2E}{\sqrt{3(1-\nu^2)}}\left(\frac{t}{R}\right)^2 \tag{2}$$

where E, ν, t, and R are respectively the Young's modulus, Poisson's ratio, thickness and radius of the shell.

Krenzke and Kiernan (1963) proposed $\gamma = 0.7$ as an empirical lower-bound knockdown factor for designing spherical shells. However, it was later argued that early experimental results were too scattered, mainly due to the lack of precise control of imperfections during fabrication (Homewood et al., 1961; Kaplan and Fung, 1954; Seaman, 1962).

There are three main issues with the knockdown factor method. First, since this method was proposed several decades ago, it does not represent current materials, design methods, and manufacturing technologies (Nemeth and Starnes, 1998). Second, currently used knockdown factors are not accurate as they were not based on systematic, extensive experimental studies (Nemeth and Starnes, 1998). In fact, recent results have shown that $\gamma = 0.2$ would be a better choice (Jimenez et al., 2016; Lee et al., 2016a,b). Third, it leads to inefficient structural designs because nearly 80% of the structure's theoretical loading capability is lost due to the reduction by γ.

An alternative approach, which has already been successful for cylindrical shells, would be to add closely spaced stiffeners in both circumferential and meridional directions of a spherical shell. However, it has been found that the buckling pressure of stiffened spherical shells is actually smaller than for unstiffened shells with the same weight (Krenzke and Kiernan, 1963). Other configurations of the stiffeners may perform better, but the number of studies of externally pressurized, stiffened spherical shells is currently quite limited (Singer et al., 2002; Ventsel and Krauthammer, 2001).

The approach proposed in this paper is fundamentally different, and was inspired by the authors' recent research on imperfection-insensitive axially loaded cylindrical shells (Ning, 2015; Ning and Pellegrino, 2013, 2015, 2017). For cylindrical shells it has been shown that extreme imperfection sensitivity can be greatly decreased or even eliminated by choosing structural shapes that break the exact axial symmetry of the cylinder. Specifically, this was done by introducing a wavy cross-section. Extending the idea of avoiding total symmetry in the design of thin shells, to enhance imperfection-sensitive behavior, this paper explores the buckling behavior and imperfection sensitivity of a range of externally pressurized sphere-like thin shells with polyhedral shapes.

This paper is organized as follows. Section 2 defines the shell geometries that are considered. The buckling behavior and imperfection sensitivity of geodesic shells are then presented in Section 3. Section 4 presents parametric studies of the behavior of stellated shells. Sections 5 and 6 discuss the mass and volume efficiencies of these shells, and Section 7 concludes the paper.

2. Geometry of near-spherical shells

The shell geometry is based on the icosahedron, chosen because it is the regular polyhedron formed of triangles that provides the closest approximation to the sphere. It encloses a larger volume than other regular polyhedra with the same circumscribed sphere. The subdivision of the icosahedron into smaller triangles leads to geodesic shells, whose geometry has already been studied extensively (Fuller, 1965; Tarnai, 1996). In addition, it has been found that inverting every pyramid of a geodesic dome into a concave shape, a transformation known as "dimpling", can significantly increase its stiffness (Kitrick, 1983; Tarnai, 1989). The choice of this particular basic shape for the present study provides access to a large design domain, controlled by only a small number of design variables.

Two families of shells are considered, *geodesic shells* and *stellated shells*. The first family provides close approximations to the sphere. The second family is based on the first, but each triangular face is replaced by a pyramid, in order to achieve a more stable design.

Please cite this article as: X. Ning, S. Pellegrino, Searching for imperfection insensitive externally pressurized near-spherical thin shells, Journal of the Mechanics and Physics of Solids (2018), https://doi.org/10.1016/j.jmps.2018.06.008