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A variational model of fracture for tearing brittle thin sheets

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Abstract

Tearing of brittle thin elastic sheets, possibly adhered to a substrate, involves a rich interplay between nonlinear elasticity, geometry, adhesion, and fracture mechanics. In addition to its intrinsic and practical interest, tearing of thin sheets has helped elucidate fundamental aspects of fracture mechanics including the mechanism of crack path selection. A wealth of experimental observations in different experimental setups is available, which has been often rationalized with insightful yet simplified theoretical models based on energetic considerations. In contrast, no computational method has addressed tearing in brittle thin elastic sheets. Here, motivated by the variational nature of simplified models that successfully explain crack paths in tearing sheets, we present a variational phase-field model of fracture coupled to a nonlinear Koiter thin shell model including stretching and bending. We show that this general yet straightforward approach is able to reproduce the observed phenomenology, including spiral or power-law crack paths in free standing films, or converging/diverging cracks in thin films adhered to negatively/positively curved surfaces, a scenario not amenable to simple models. Turning to more quantitative experiments on thin sheets adhered to planar surfaces, our simulations allow us to examine the boundaries of existing theories and suggest that homogeneous damage induced by moving folds is responsible for a systematic discrepancy between theory and experiments. Thus, our computational approach to tearing provides a new tool to understand these complex processes involving fracture, geometric nonlinearity and delamination, complementing experiments and simplified theories.

Keywords: variational model, tearing, fracture, thin sheets, subdivision surface.

1. Introduction

Thin elastic sheets are very common in nature and technology. In addition to an in-plane mode of fracture, thin sheets exhibit tearing, a situation in which cracks propagate driven by out-of-plane loading. Tearing a thin sheet is a very common experience in our daily life when we peel a piece of fruit or open a package. We lack, however, a complete theoretical understanding of this phenomenon, which challenges classical theories of fracture. In classical fracture mechanics, various crack path selection criteria have been successful in predicting crack propagation in bulk brittle materials, including the maximum hoop stress criterion (Erdogan and Sih, 1963), the principle of local symmetry (Cotterell, 1965; Goldstein and Salganik, 1974), the minimum strain energy density (Sih, 1974), or maximum energy release rate (Wu, 1978; Palaniswamy and Knauss, 1978). While these different criteria are very similar, or even equivalent, for bulk isotropic materials, it is far from obvious how to generalize some of them to a brittle, possibly anisotropic, thin sheet (Takei et al., 2013; Roman, 2013; Ibarra et al., 2016). For instance, the principle of local symmetry

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