# Steady-state crack growth in polymer gels: A linear poroelastic analysis

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A B S T R A C T

Based on a linear poroelastic formulation, we present an asymptotic analysis of the crack tip fields for steady-state crack growth in polymer gels. A finite element method is then developed for numerical analysis. A semi-infinite crack in a long strip specimen subject to plane-strain loading is studied in detail. The crack-tip fields from the numerical analysis agree with the asymptotic analysis with a poroelastic stress intensity factor, which is generally smaller than the stress intensity factor predicted by linear elasticity due to poroelastic shielding. Similarly, the crack-tip energy release rate calculated by a modified *J*-integral method is smaller than the applied energy release rate. The size of the poroelastic crack-tip field (or *K*-field) is characterized by a diffusion length scale that is inversely proportional to the crack speed. For relatively fast crack growth, the diffusion length is small compared to the strip thickness (small-scale diffusion), and the poroelastic *K*-field transitions to an elastic *K*-field at a distance proportional to the diffusion length. In this case, the crack-tip energy release rate decreases with increasing crack speed under the same loading condition. For relatively slow crack growth, the diffusion length is greater than the strip thickness (large-scale diffusion), and the poroelastic crack-tip field is confined and transitions to a one-dimensional diffusion zone ahead of the crack tip. In this case, the energy release rate increases with increasing crack speed. Both immersed and not-immersed crack face conditions are considered. Under the same loading conditions, the poroelastic stress intensity factor and the modified *J*-integral are higher for the immersed case than not-immersed, but they approach the same values at the fast and slow crack limits. In general, if the crack-tip energy release rate is taken to be the intrinsic fracture toughness of the gel, the applied energy release rate as the apparent fracture toughness is greater due to energy dissipation associated with solvent diffusion, which is referred to as poroelastic toughening. It is proposed that the modified *J*-integral can be used to determine the intrinsic fracture toughness of the gel in experiments, which may or may not depend on crack speed. Moreover, the present results are found to be qualitatively consistent with previous experiments on the effects of solvent viscosity and crack-tip soaking.

## 1. Introduction

Polymer gels consist of crosslinked polymer chains and solvent molecules (e.g., water). The mechanical properties of polymer gels are similar to soft biological tissues such as tendons, ligaments, cartilage, muscles, skin, and nerves. As a result,

polymer gels have been widely used in biomedical applications (Drury and Mooney, 2003; Langer, 2006; Peppas et al., 2006). More recently, polymer gels have also been exploited as a class of soft active materials with sensing and actuating properties in the development of soft machines and soft robotics (Calvert, 2009; Suo, 2012; Yuk et al., 2017). These applications have motivated development of tough hydrogels (Gong et al., 2003; Sun et al., 2012; Zhao, 2014) as well as fundamental studies on fracture mechanics of polymer gels (Wang and Hong, 2012; Hui et al., 2013; Lefranc and Bouchaud, 2014; Bouklas et al., 2015a; Long and Hui, 2016; Noselli et al., 2016; Long et al., 2016; Tang et al., 2017; Yang and Lin, 2018; Mao and Anand, 2018).

To enhance the facture toughness of polymer gels, a number of energy dissipation mechanisms have been employed (Zhao, 2014; Long and Hui, 2016), including both rate-insensitive damage mechanisms and rate-dependent mechanisms such as viscoelastic and poroelastic effects. The present study focuses on the poroelastic effects only, to investigate the effects of solvent diffusion on steady-state crack growth. Previously, Hui et al. (2013) studied the short time transient stress fields near the tip of a stationary crack in a poroelastic solid, based on the theory of linear poroelasticity. Bouklas et al. (2015a) proposed a modified J-integral method for calculating energy release rate of quasi-static crack growth in gels with large deformation and solvent diffusion. With a nonlinear, transient finite element method (Bouklas et al., 2015b), they studied the effects of solvent diffusion on the crack-tip fields and the energy release rate for stationary cracks in polymer gels. Noselli et al. (2016) presented a theoretical and numerical study on quasi-static, steady-state crack growth in polymer gels, also based on linear poroelasticity. They predicted a poroelastic toughening effect that is independent of crack velocity and then proposed a poroelastic cohesive zone model for the dependence of effective toughness on crack velocity. In the present study, we develop an asymptotic analysis of the crack-tip fields for steady-state crack growth in polymer gels based on linear poroelasticity and numerically show that the crack-tip energy release rate depends on crack velocity in a long strip specimen. The results are consistent with Noselli et al. (2016) at the limit of "fast" crack growth when the characteristic steady-state diffusion length scale is much smaller than the specimen size (small-scale diffusion).

Experimental measurements have reported a wide range of fracture toughness for polymer gels, from $\sim 1 \, \mathrm{J/m^2}$ for gelatin and agar gels (Forte et al., 2015; Lefranc and Bouchaud, 2014) to $\sim 9000 \, \mathrm{J/m^2}$ for a hybrid alginate-polyacrylamide gel (Sun et al., 2012). However, as noted by Long and Hui (2016), most of these measurements were interpreted by assuming that the gels are purely elastic or rate independent despite the fact that the gel typically contains over 90 wt% of water (or other solvents). Rate-dependent fracture of polymer gels may result from dynamic damage processes at the crack tip, viscoelasticity and poroelasticity. For example, Lefranc and Bouchaud (2014) attributed the crack velocity dependent toughness of agar gels to viscous chain pull-out or stress accelerated chain dynamics, both localized at the crack tip region. Wang and Hong (2012) suggested a visco-poroelastic mechanism for the delayed fracture of polymer gels that was observed in the experiments of Bonn et al. (1998). Moreover, in an experimental study on crack growth in gelatin gels, Baumberger et al. (2006) found that increasing solvent viscosity slowed crack growth, soaking the crack tip with solvent increased gel fragility, and in general, the effective fracture energy increased with crack velocity (so-called "velocity toughening"). In the present study we show that these observations are qualitatively consistent with the poroelastic model of steady-state crack growth.

The remainder of this paper is organized as follows. Section 2 presents a linear poroelastic formulation for polymer gels derived from a nonlinear theory. Section 3 describes a steady-state crack growth model. In Section 4, we develop an asymptotic analysis for the steady-state crack tip fields in a linearly poroelastic material. A finite element method is presented in Section 5, and the numerical results are discussed in Section 6. Two limiting cases (fast and slow crack growth) are presented in appendices. Section 7 concludes the present study with a brief summary.

## 2. A linear poroelastic formulation

The elastic deformation of the polymer network and migration of solvent molecules are generally coupled in a polymer gel. Both linear and nonlinear theories have been developed for gels. Although in general a nonlinear theory is necessary to describe large deformation of gels, a linear theory may be used as an approximation with the advantage of having possible analytical solutions. It has been shown that the generally nonlinear field theory for polymer gels (e.g., Hong et al., 2008) can be linearized for small deformation from a swollen state, leading to a linear poroelastic formulation (Bouklas and Huang, 2012). Here we summarize the linear formulation, with which an asymptotic analysis is presented for the steady-state crack-tip fields in Section 4 and a corresponding finite element method is developed in Section 5.

Let the gel be stress free and isotropically swollen in the initial state, where the solvent in the gel has a chemical potential, $\mu = \mu_0$, in equilibrium with an external solution of the same chemical potential. Relative to the dry state of the polymer network, the linear swelling ratio $\lambda_0$ of the gel in the initial state can be related to the chemical potential $\mu_0$ as

$$N\Omega\left(\frac{1}{\lambda_0} - \frac{1}{\lambda_0^3}\right) + \ln\frac{\lambda_0^3 - 1}{\lambda_0^3} + \frac{1}{\lambda_0^3} + \frac{\chi}{\lambda_0^6} = \frac{\mu_0}{k_B T}, \tag{2.1}$$

where $N$ is the effective number density of polymer chains in the dry state, $\Omega$ is the volume of the solvent molecule, $\chi$ is the Flory–Huggins parameter for polymer–solvent interactions, $k_B T$ is the temperature in the unit of energy with the Boltzmann constant $k_B$ and the absolute temperature $T$. As in previous studies (Hong et al., 2009; Kang and Huang, 2010), the nonlinear relation is derived from the Flory–Rehner theory of polymer gels (Flory, 1953).