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Modified multiplicative decomposition model for tissue growth: Beyond the initial stress-free state

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ABSTRACT

The multiplicative decomposition model is widely employed for predicting residual stresses and morphologies of biological tissues due to growth. However, it relies on the assumption that the tissue is initially in a stress-free state, which conflicts with the observations that any growth state of a biological tissue is under a significant level of residual stresses that helps to maintain its ideal mechanical conditions. Here, we propose a modified multiplicative decomposition model in which the initial state (or reference configuration) of a biological tissue is endowed with a residual stress instead of being stress-free.

Releasing theoretically the initial residual stress, the initially stressed state is first transmitted into a virtual stress-free state, thus resulting in an initial elastic deformation. The initial virtual stress-free state subsequently grows to another counterpart with a growth deformation, and the latter is further integrated into its natural configuration of a real tissue with an excessive elastic deformation that ensures tissue compatibility. With this decomposition, the total deformation arising during growth may be expressed as the product of elastic deformation, growth deformation and initial elastic deformation, while the corresponding free energy density should depend on the initial residual stress and the total deformation. Three key issues including the explicit expression of the free energy density, the predetermination of the initial elastic deformation, and the initial residual stress are addressed.

Finally, we consider a tubular organ as a representative example to demonstrate the effects of the proposed initial residual stress on stress distribution and on shape formation through an incremental stability analysis. Our results suggest that the initial residual stress exerts a major influence on the growth stress and the morphology of biological tissues. The model bridges the gap between any two growth states of a biological tissue that is endowed with a certain level of residual stresses.

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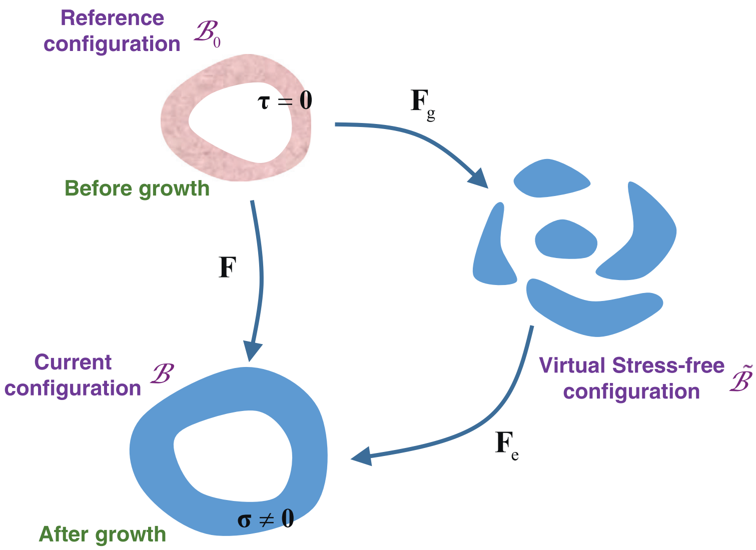

Fig. 1. The classical multiplicative decomposition proposed by Rodriguez.

1. Introduction

It has long been recognized that growth, death and all other bio-behaviors of living matter are controlled by a combination of genetic and epigenetic factors including biochemistry, bioelectricity and biomechanics (Cowin, 2004; Fung, 2013). One of the most accepted biomechanical epigenetic factor in living matter may be its internal mechanical stress, which is also called residual stress in unloaded conditions (Cowin, 2006; Eskandari and Kuhl, 2015; Hosford, 2010; Schajer, 2013). It exists in all real living matter such as ripe fruits, tree trunks, blood vessels or solid tumors, and is generally induced by non-uniform plastic deformation, surface modification, material phase changes and/or density changes (Chen and Eberth, 2012; Chuong and Fung, 1983; Schajer, 2013; Stylianopoulos et al., 2012). Though these residual stresses are locally self-equilibrating in mechanics, they still serve some special functions, as they, for example, influence the morphogenesis, growth rate and internal mechanical condition of bio-tissues (e.g. Ben Amar and Goriely, 2005; Fung, 1991; Li et al., 2011b; Taber, 1998).

The first presentation on the relationship between stress and bio-behavior goes back to the German anatomist and surgeon Wolff (1893) who showed that healthy bone creates structural adaptation where external loads are placed. Then Roux (1894) proposed the functional adaptation concept that stress should be regarded as a functional stimulus to growth and remodeling. Later, Fung and collaborators (Chuong and Fung, 1986; Fung, 1991) used the opening angle method to quantify residual stress in arteries.

An explanation on the origin of residual stresses in living bio-tissues was first presented theoretically by Rodriguez et al. (1994) via the multiplicative decomposition (MD) method. They showed that residual stresses in a bio-tissue are created by heterogeneous growth and can be calculated from a given growth gradient tensor. The constrained growth deformation is decomposed into unconstrained growth deformation and pure elastic deformation (Fig. 1) with the relation $\mathbf{F} = \mathbf{F}_e\mathbf{F}_g$, where $\mathbf{F}$ is the total deformation, $\mathbf{F}_e$ the pure elastic deformation, and $\mathbf{F}_g$ the growth deformation. From a modeling standpoint, this explanation is concise but powerful enough to predict the growth-induced residual stresses. The MD model was subsequently widely employed to solve many biomechanical problems related to growth process (e.g. Balbi et al., 2015; Du and Lü, 2017; Li et al., 2011b; Lü and Du, 2016; Stylianopoulos et al., 2012; Wang et al., 2017).

One prerequisite of the MD model is that the reference configuration must be a *stress-free state*, which ensures that the growth process is under the unconstrained condition. To study the growth process of bio-tissues starting from an arbitrary stage, the initial reference state should be properly defined. As evidenced by cutting experiments, many bio-tissues still exhibit large amounts of residual stresses even when the external loads are removed, as seen with a cut scallion, a duck heart or liver, see Fig. 2, and also with cut arteries or weasands (Li et al., 2011b).

An approximate method to access the stress-free state is by cutting the material to remove constraints from surrounding tissues, which is the basic idea behind the *opening angle method* (e.g. Chuong and Fung, 1986; Gower et al., 2015; Schajer, 2013). However, the extent to which residual stresses can be released depends significantly on the number and direction of the cuts (Fig. 2b). In effect, an entirely stress-free state for a real living body can only be accomplished by an infinite number of cuts to release all residual stresses held by the neighboring regions. In practice, this ultimate discrete state is impossible to reach for real living matter (Schajer, 2013). This suggests that stress-free state may not be achievable for real