Journal of the Mechanics and Physics of Solids 117 (2018) 37–53

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

On the band gap universality of multiphase laminates and its applications

Ben Lustig, Gal Shmuel*

Faculty of Mechanical Engineering, Technion-Israel Institute of Technology, Haifa, 32000, Israel

ARTICLE INFO

Article history:
Received 21 March 2018
Accepted 13 April 2018
Available online 21 April 2018

Keywords:
Composite
Multiphase laminate
Bloch-Floquet analysis
Band gap
Phononic crystal
Wave propagation
Finite deformations

ABSTRACT

The band structure of a periodic medium describes which wave frequencies, termed gaps, it filters out, depending on the medium composition. Shmuel and Band (2016) discovered that all infinite band structures of two-phase laminates impinged by normal waves are remarkably encapsulated in a finite geometric object, independently of the specific laminate composition. We here unveil a generalized object that encapsulates the band structures of all *multiphase* laminates impinged by normal waves. The merit of such a universal object is more than mathematical beauty—it establishes a platform for unprecedented characterization of the band structure. We specifically exploit it to rigorously determine the density of the gaps in the spectrum, and prove it exhibits universal features. We further utilize it to formulate optimization problems on the gap width and develop a simple bound. Using this framework, we numerically study the dependency of the gap density and width on the impedance and number of phases. In certain settings, our analysis applies to nonlinear multiphase laminates, whose band diagram is tunable by pre-deformations. Through simple examples, we demonstrate how the universal object is useful for tunability characterization. Our insights may establish a step towards engineering filtering devices according to desired spectral properties.

© 2018 Elsevier Ltd. All rights reserved.

1. Introduction

Periodicity in a transmission medium of waves renders their propagation frequency dependent, even when the medium pointwise properties are not (Hussein et al., 2014). The resultant propagation can exhibit exotic or *metamaterial* characteristics, such as negative refraction and wave steering (Lu et al., 2009; Zelhofer and Kochmann, 2017). In addition to the intriguing nature of metamaterials, they have functional potential in applications such as cloaking and superlensing (Colquitt et al., 2014; Milton et al., 2006; Pendry, 2000).

Laminates—the media addressed in this paper—have the simplest periodicity, as their properties change only along one direction. Surprisingly, new results on their dynamics are still reported by ongoing research, *e.g.*, metamaterial behavior of laminates (Bigoni et al., 2013; Nemat-Nasser, 2015; Srivastava, 2016; Willis, 2016), field patterns in laminates with time dependent moduli (Milton and Mattei, 2017), and dynamic homogenization of laminates (Joseph and Craster, 2015; Nemat-Nasser and Srivastava, 2011; Nemat-Nasser et al., 2011; Sheinfux et al., 2014; Srivastava and Nemat-Nasser, 2014).

We are concerned with the most familiar phenomenon: annihilation of waves at certain frequencies and corresponding emergence of a band-gap structure in the infinite spectrum (Kushwaha et al., 1993; Sigalas and Economou, 1992). The band

* Corresponding author.
E-mail address: meshmuel@technion.ac.il (G. Shmuel).

https://doi.org/10.1016/j.jmps.2018.04.008
0022-5096/© 2018 Elsevier Ltd. All rights reserved.

Fig. 1. Laminates with different periodic cells. The band structure describes which frequency *bands* are of propagating waves (middle laminate), and which frequencies belong to *gaps* of decaying waves (right laminate), depending on the periodic cell composition. In this work, we unveil a generalized object that encapsulates the infinite band structures of all *multiphase* laminates impinged by normal waves, independently of the specific composition.

structure describes which frequency *bands* are of propagating waves, and which frequencies belong to *gaps* of decaying waves. The range of these gaps varies from one laminate to another, as function of composition, *i.e.*, the thickness and mechanical properties of the comprising layers (Fig. 1). These gaps are not only physically interesting, they may also be exploited to absorb undesired vibrations and filter noise (*e.g.*, the realizations of Babaee et al., 2016; Matlack et al., 2016, using more complex periodicities). Hence, a complete characterization of band structures and their relation to the laminate composition is also of applicational importance. For waves at normal incident angle, Shmuel and Band (2016) discovered that all band structures of laminates with two alternating layers are remarkably derived from a finite geometric object, *independently* of each layer thickness and specific physical properties. They used this universality with respect to the layer properties to obtain unprecedented characterization of the band structures. Specifically, they rigorously derived the maximal width, expected width, and density of the gaps, *i.e.*, the relative width of the gaps within the entire spectrum. Finally, Shmuel and Band (2016) conjectured that such universality also exists for laminates made of an arbitrary number of phases. In what follows, we unveil a finite generalized object—a compact manifold—that encapsulates the band structures of all multiphase laminates impinged by normal waves. We employ it to answer several interesting questions, *e.g.*, can the gap density be increased by adding more phases? If so, what are the optimal compositions that maximize it? Can we enlarge specific gaps in the same way? Our answers and insights may establish a step towards finding their counterparts in more complex systems and, in turn, engineering filtering devices according to desired spectral properties.

The bulk of our analysis is presented in the framework of linear infinitesimal elasticity; in the sequel we show that in certain settings it extends to finite elasticity. Specifically, we show that our analysis applies for incremental waves propagating in non-linear multiphase laminates subjected to piecewise-constant finite deformations. Similarly to the case of finitely deformed two-phase laminates, the resultant band diagram is tunable by the static finite deformation (Shmuel and Band, 2016, *cf.*, Zhang and Parnell, 2017); through simple examples, we demonstrate how the universality of our representation is useful for characterizing this tunability.

We present our results in the following order. Firstly, in Sec. 2 we revisit the derivation of the dispersion relation for multiphase laminates, from which the band structure is evaluated. Sec. 3 contains our theory; therein, we show that all infinite band structures of multiphase laminates are encapsulated in a compact universal manifold, whose dimensionality equals the number of layers in the periodic cell. We find that the gap density is the volume fraction of a universal submanifold, derive a closed-form expression for the submanifold boundary, and hence for the gap density. We further employ the new framework to provide a simple bound on the gapwidth and formulate corresponding optimization problems. Sec. 4 employs our formulation to answer the questions posed earlier, via parametric investigation of the compact manifold. Sec. 5 details how our theory extends to non-linear multiphase laminates of tunable band diagrams, and demonstrates its application for characterizing this tunability. Finally, we summarize our results in Sec. 6.

2. Wave propagation in multiphase laminates

2.1. Dispersion relation

The solution to the problem of wave propagation in periodic laminates is well-known (Rytov, 1956); the topic receives renewed attention recently, owing to its applications in the context of metamaterials (Srivastava, 2016; Willis, 2016). For the reader convenience, this Sec. concisely recapitulates the formulation for multiphase laminates (Lekner, 1994), in the framework of linear elasticity.

A laminate is made of an infinite repetition of a unit cell comprising N layers. We denote the thickness, mass density, and Lamé coefficients of the n^{th} layer by $h^{(n)}$, $\rho^{(n)}$, $\lambda^{(n)}$ and $\mu^{(n)}$, respectively. We consider plane waves propagating normal to the layers at frequency ω. The equations of motion are satisfied by a displacement field which can be written in each

Download English Version:

https://daneshyari.com/en/article/7177402

Download Persian Version:

https://daneshyari.com/article/7177402

Daneshyari.com