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Modal nudging in nonlinear elasticity: Tailoring the elastic post-buckling behaviour of engineering structures

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ABSTRACT

The buckling and post-buckling behaviour of slender structures is increasingly being harnessed for smart functionalities. Equally, the post-buckling regime of many traditional engineering structures is not being used for design and may therefore harbour latent load-bearing capacity for further structural efficiency. Both applications can benefit from a robust means of modifying and controlling the post-buckling behaviour for a specific purpose. To this end, we introduce a structural design paradigm termed *modal nudging*, which can be used to tailor the post-buckling response of slender engineering structures without any significant increase in mass. Modal nudging uses deformation modes of stable post-buckled equilibria to perturb the undeformed baseline geometry of the structure imperceptibly, thereby favouring the seeded post-buckling response over potential alternatives. The benefits of this technique are enhanced control over the post-buckling behaviour, such as modal differentiation for smart structures that use snap-buckling for shape adaptation, or alternatively, increased load-carrying capacity, increased compliance or a shift from imperfection sensitivity to imperfection insensitivity. Although these concepts are, in theory, of general applicability, we concentrate here on planar frame structures analysed using the nonlinear finite element method and numerical continuation procedures. Using these computational techniques, we show that planar frame structures may exhibit isolated regions of stable equilibria in otherwise unstable post-buckling regimes, or indeed stable equilibria entirely disconnected from the natural structural response. In both cases, the load-carrying capacity of these isolated stable equilibria is greater than the natural structural response of the frames. Using the concept of *modal nudging* it is possible to "nudge" the frames onto these equilibrium paths of greater load-carrying capacity. Due to the scale invariance of modal nudging, these findings may impact the design of structures from the micro- to the macro-scale.

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1. Introduction

In structural mechanics, nonlinearities are often viewed as failure modes. In the case of material nonlinearity, where plastic deformations irreversibly change the constitutive behaviour of the material, avoiding nonlinearity is a good heuristic. On the other hand, reversible nonlinearities can be employed for additional functionality. As a typical example, elastic snap-buckling of a shallow arch can be viewed as a failure mode if the application is a footbridge, but can also be treated as a valuable mechanism for rapid and reversible shape-change in applications such as energy-harvesting devicses (Harne and Wang, 2013).

In this sense, while buckling is historically viewed as a failure event with little practical significance, exploiting buckling for smart functionality has become a prevalent theme in the literature (Hu and Burgueño, 2015a; Reis, 2015). The disciplines of micro-electromechanical systems (MEMS) (Andò et al., 2010; Bogue, 2013; Jog and Patil, 2016; Tsai and Sue, 2007); deployable, morphing and compliant structures (Arena et al., 2017; Cox et al., 2018; Gomez et al., 2017; Groh and Pirrera, 2018a; Pirrera et al., 2012; Previtali et al., 2011; Sofla et al., 2010); meta-materials (Bertoldi et al., 2010; Florijn et al., 2014; Mullin et al., 2007; Overvelde et al., 2012); and energy harvesters (Harne and Wang, 2013) are all leading the way in embracing buckling as a design tool.

Many smart applications employ bistable components for additional functionality. In this case, the buckling behaviour is binary and no additional control, beyond the applied load, is required to transition between two configurations. Alternatively, shape-adaptive components can be designed to feature a number of different stable post-buckled configurations, and in this case, a means of modal differentiation is required (Groh and Pirrera, 2018b). This is especially the case, because structures that can attain a larger number of stable configurations, such as cylindrical shells, suffer from severe imperfection sensitivity, and hence, prediction and control of the observed behaviour is challenging.

Equally, traditional engineering structures can benefit from controlling the post-buckling behaviour. A major goal of any post-buckling analysis is to differentiate between post-critical behaviour that is stable and progressive, or unstable and catastrophic. In case of the former, the structure continues to take load beyond the first instability point, but often with reduced rigidity. Furthermore, the structure can have more than one stable post-buckling response, with no guarantee that the naturally observed behaviour leads to the greatest load-carrying capacity or compliance before failure. In case of the latter, the initially unstable post-critical behaviour typically stabilises deeper into the post-critical regime, but once the structure snaps into this configuration, the load-carrying capability of the structure has often been reduced or irreversible material nonlinearity has occurred. Furthermore, the sharp two-thirds power law of the cusp catastrophe, which governs the relation between buckling load and geometric imperfections, implies a pronounced and detrimental sensitivity to imperfections (Thompson and Hunt, 1973). The combination of imperfection sensitivity and the stochastic nature of imperfections leads to uncertainty during design (Arbocz and Hol, 1995), which means that imperfection-sensitive structures, such as cylindrical shells under axial compression, are designed with empirically derived and often conservative knock-down factors (Jiménez et al., 2017).

In this vein, this paper explores the potential of controlling structural nonlinearities to tailor the post-buckling response of engineering structures for a specific purpose. Predominant in this field is the work by Mang et al. (2006) and Schranz et al. (2006) who investigated potential means of transforming imperfection-sensitive designs into imperfection-insensitive ones. Their research shows that this shift can be achieved by (i) altering the thickness of the structure; (ii) attaching auxiliary members (*e.g.* springs); and (iii) varying the geometry of the structure. Although this approach can be highly effective, it has drawbacks if certain geometrical space constraints need to be imposed, or when it is not possible to fix additional components to the structure. Ning and Pellegrino (2015) showed that the imperfection-sensitive cylindrical shell under axial compression can be transformed into an imperfection-insensitive design by changing the cross-sectional topology. In doing so, the cross-section is transformed from circular to wavy, potentially rendering the new structure geometrically less useful for operation in service.

White and Weaver (2016) introduced an interesting concept for imperfection-insensitive cylindrical shells by using the stiffness-tailoring capacity of fibre-reinforced plastic materials. By means of an optimisation framework, White and Weaver (2016) showed that laminated composites with curvilinearly varying fibre paths can be used to tailor the elastic modulus across cylindrical panels such that there is no degradation in axial stiffness in the post-buckling regime. In this manner, the typically unstable “shell-like” post-buckling response can be transformed into a more benign stable “plate-like” response. Similarly, Wu et al. (2018) used the technology of fibre-steering to tailor both the fibre orientation and thickness of composite panels under compression to optimise to an effectively “buckle-free” structure with negligible loss of axial stiffness in the post-critical regime.

Finally, Burgueño and co-workers have published extensively on tailoring the post-buckling behaviour of slender structures for smart applications. For example, the mode-jumping characteristics (number of mode jumps, load drops during mode jumping, hysteretic energy dissipation, *etc.*) in the post-buckling regime of axially compressed cylindrical shells may be controlled by patterned stiffness distributions, lateral constraints and laminate stacking sequence variations (Burgueño et al., 2014). Similarly, the elastic post-buckling response in terms of initial and final stiffness can also be tailored using modal superpositions of buckling modes seeded as initial imperfections (Hu and Burgueño, 2017). This approach is also valuable for reducing the imperfection sensitivity of axially compressed cylindrical shells (Hu et al., 2014; Hu and Burgueño, 2015b).