Journal of the Mechanics and Physics of Solids 115 (2018) 195–207

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Three-dimensional fracture instability of a displacement-weakening planar interface under locally peaked nonuniform loading

Koji Uenishi

Department of Aeronautics and Astronautics, The University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113–8656, Japan

ARTICLE INFO

Article history:
Received 19 October 2017
Revised 9 February 2018
Accepted 18 March 2018
Available online 19 March 2018

Keywords:
Fracture mechanisms
Fracture nucleation
Constitutive behavior
Crack mechanics
Energy methods

ABSTRACT

We consider stability of fracture on a three-dimensional planar interface subjected to a loading stress that is locally peaked spatially, the level of which increases quasi-statically in time. Similar to the earlier study on the two-dimensional case (Uenishi and Rice, 2003; Rice and Uenishi, 2010), as the loading stress increases, a crack, or a region of displacement discontinuity (opening gap in tension or slip for shear fracture), develops on the interface where the stress is presumed to decrease according to a displacement-weakening constitutive relation. Upon reaching the instability point at which no further quasi-static solution for the extension of the crack on the interface exists, dynamic fracture follows. For the investigation of this instability point, we employ a dimensional analysis as well as an energy approach that gives a Rayleigh–Ritz approximation for the dependence of crack size and maximum displacement discontinuity on the level and quadratic shape of the loading stress distribution. We show that, if the linear displacement-weakening law is applied and the crack may be assumed of an elliptical form, the critical crack size at instability is independent of the curvature of the loading stress distribution and it is of the same order for all two- and three-dimensional cases.

1. Introduction

Understanding the physical process of destabilization of partially fractured interfaces in solids is of crucial importance not only in the field of the mechanics of solid materials (to name a few, Ranjith and Rice, 2001; Suo et al., 1992; Wang et al., 2015) but also in the dynamic study of an earthquake, often considered as development of shear fracture on an interface (geological fault) (e.g. Bizzarri, 2010; Galis et al., 2015; Shi et al., 2008). Typical models to examine the mechanical stability of an elastic structure include a quasi-statically growing cohesive crack with constitutive laws that are dependent on displacement discontinuity (Bazant and Li, 1995a; 1995b; Li and Liang, 1993), where the stress is specified as a decreasing function of the crack opening displacement (displacement-weakening or tension-softening constitutive relation), and mathematical spectral analyses related to earthquakes can show the initiation of dynamic anti- or in-plane slip instabilities of a slip-weakening (analogous to displacement-weakening, but for shear fracture) fault that is *a priori* of fixed length and located in a homogeneous, linear elastic medium and uniformly preloaded up to the frictional threshold (Campillo and Ionescu, 1997; Dascalu et al., 2000; Favreau et al., 1999; Ionescu and Campillo, 1999). By deriving eigenvalue problems, the spectral investigation has given an analytical expression of the displacement discontinuity and divided that displacement

E-mail address: uenishi@dyn.t.u-tokyo.ac.jp

https://doi.org/10.1016/j.jmps.2018.03.012

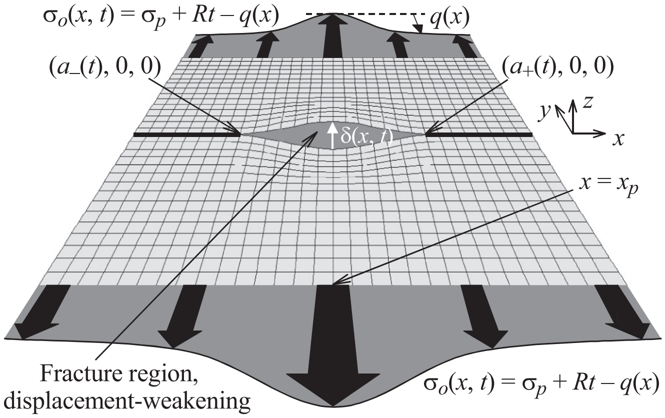

Fig. 1. A schematic displacement field related to tensile fracture (mode I) in an infinite, homogeneous, isotropic, linear elastic two-dimensional space. The nonuniform loading tensile stress $\sigma_o(x, t)$ is locally peaked in space and varies quasi-statically with time t, at rate R. The stress inside the fracture region (crack) drops according to a displacement-weakening law (Fig. 2). Similarly, we can define the two-dimensional problems for in- and anti-plane shear loading (modes II and III), e.g. for tectonic loading acting on an earthquake fault.

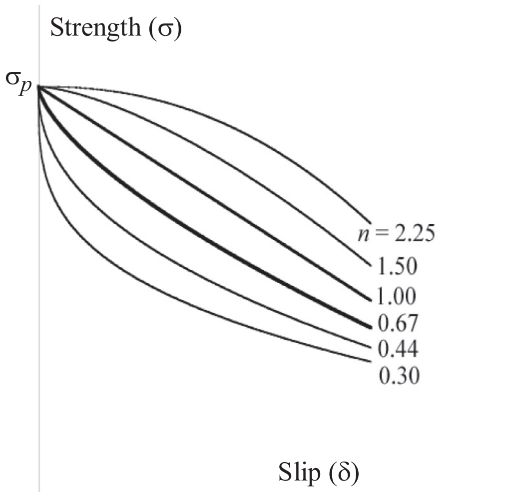

Fig. 2. The nonlinear, power-law displacement-weakening constitutive relation. The stress inside the fracture region on the interface obeys the relation given by $\sigma = \sigma_p - W\delta^n$, where W and n are positive constants. The initial gradient of the curve, $d\sigma/d\delta(\delta = 0^+)$, is zero if $n > 1$, $-W(< 0)$ if $n = 1$ (linear displacement-weakening) and $-\infty$ if $n < 1$ (modified after Rice and Uenishi (2010)).

discontinuity into two parts: the solution associated with positive ("dominant part") and negative ("wave part") eigenvalues. The dominant part exponentially grows with time and governs the development of the instability and the wave part quickly becomes negligible as instability develops. The role of displacement-weakening slope on the duration of the quasi-static fracture nucleation phase as well as the critical crack length has been evaluated in these slip analyses. However, the physical meaning of the assumption of the (*a priori*) fixed fault length and uniform loading was not explained in the work, and more realistic treatment of initially quasi-static fracture growth (in size) before destabilization and dynamic propagation was needed for the stability analysis of fracture on an interface (Uenishi and Rice, 2003).

Therefore, in the earlier study (Rice and Uenishi, 2010; Uenishi and Rice, 2003), we addressed the quasi-static fracture behaviors of linear and nonlinear displacement-weakening interfaces in the two-dimensional context and quantitatively assessed the critical crack length related to interface instabilities and the following dynamic fracture. We considered interface fracture in an infinite, homogeneous, isotropic linear elastic medium that is subjected to a nonuniform, locally peaked loading stress (see Fig. 1),

$$\sigma_o(x,t) = \sigma_p + Rt - q(x), \tag{1}$$

where the interface coincides with the $x - z$ plane ($y = 0$) of a Cartesian coordinate system xyz, σ_p is the tensile (for mode I) or shear (for modes II and III) strength of the interface, and R (> 0 if the stress increases with time t) is the loading rate of the increasing stress. The function representing the exact shape of the loading stress distribution, $q(x)$, satisfies $q(x) > 0$ for $x \neq x_p$ and $q(x_p) = 0$. The time $t = 0$ corresponds to the one when the peak value of the loading stress first reaches σ_p at x_p