Journal of the Mechanics and Physics of Solids 114 (2018) 209–218

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

An approximate JKR solution for a general contact, including rough contacts

M. Ciavarella

Politecnico di BARI, Center of Excellence in Computational Mechanics, Viale Gentile 182, Bari 70126, Italy

ARTICLE INFO

Article history:
Received 15 December 2017
Revised 28 February 2018
Accepted 3 March 2018
Available online 5 March 2018

Keywords:
Contact
Roughness
Adhesion
JKR model
Persson's theory

ABSTRACT

In the present note, we suggest a simple closed form approximate solution to the adhesive contact problem under the so-called JKR regime. The derivation is based on generalizing the original JKR energetic derivation assuming calculation of the strain energy in adhesiveless contact, and unloading at constant contact area. The underlying assumption is that the contact area distributions are the same as under adhesiveless conditions (for an appropriately increased normal load), so that in general the stress intensity factors will not be exactly equal at all contact edges. The solution is simply that the indentation is $\delta = \delta_1 - \sqrt{2wA'/P''}$ where w is surface energy, δ_1 is the adhesiveless indentation, A' is the first derivative of contact area and P'' the second derivative of the load with respect to δ_1. The solution only requires macroscopic quantities, and not very elaborate local distributions, and is exact in many configurations like axisymmetric contacts, but also sinusoidal waves contact and correctly predicts some features of an ideal asperity model used as a test case and not as a real description of a rough contact problem. The solution permits therefore an estimate of the full solution for elastic rough solids with Gaussian multiple scales of roughness, which so far was lacking, using known adhesiveless simple results. The result turns out to depend only on rms amplitude and slopes of the surface, and as in the fractal limit, slopes would grow without limit, tends to the adhesiveless result – although in this limit the JKR model is inappropriate. The solution would also go to adhesiveless result for large rms amplitude of roughness h_{rms}, irrespective of the small scale details, and in agreement with common sense, well known experiments and previous models by the author.

1. Introduction

Exact solution to adhesive problems are very scarce. Bradley (1932) and Derjaguin (1934) obtained the adhesive force between two *rigid* spheres, equal to $2\pi Rw$, where w is the work of adhesion, and R is the radius of the sphere. Then, JKR (Johnson et al., 1971) developed the first exact theory for elastic bodies, namely spheres, assuming adhesive forces occur entirely within the contact area, obtaining 3/4 of the Bradley pull-off value, and independence on the elastic modulus which seem to indicate that the result would be corresponding to the rigid Bradley limit. The result was even more surprising when Derjaguin–Muller–Toporov (DMT) developed their elastic theory (Derjaguin et al., 1975) which seemed to indicate the same pull-off value of Bradley rather that JKR. Tabor brilliantly solved the dilemma, indicating transition from rigid to JKR

E-mail addresses: Mciava@poliba.it, michele.ciavarella@poliba.it

https://doi.org/10.1016/j.jmps.2018.03.005
0022-5096/

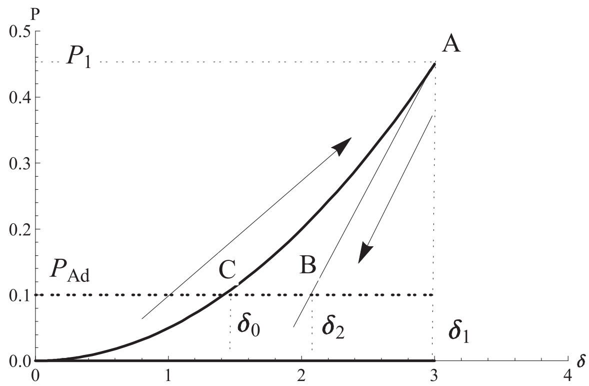

Fig. 1. The loading scenario. (i) "repulsive" loading without adhesive forces until the contact area is a given value (load path $\overline{OA}$ as in the original JKR paper); (ii) rigid-body displacement at constant total contact area A (load path $\overline{AB}$ as in the original JKR paper).

depends on the Tabor parameter (Tabor, 1977)

$$\mu = \left(\frac{Rw^2}{E^{*2} a_0^3} \right)^{1/3} \tag{1}$$

where a_0 is the range of attraction of adhesive forces, close to atomic distance for crystals, and E^* the plane strain elastic modulus.

The JKR regime therefore is only valid for large Tabor parameters (especially if instability at jump-into contact is required accurately, see Ciavarella et al., 2017). JKR permits to find many solutions easily by superposition of contact and crack solutions (see Johnson, 1995), whereas the original JKR energetic method has been less popular except of course the adhesion problem can be formulated in elaborate numerical algorithms by minimization methods (Carbone et al., 2015). Particularly the problem of rough surfaces has seen significant effort in the last 40 years or so (Afferrante et al., 2015; Ciavarella, 2015, 2017a, 2017b; Ciavarella et al., 2017; Ciavarella and Papangelo, 2018a, 2018b, 2018c; Ciavarella et al., 2018; Fuller and Tabor, 1975; Pastewka and Robbins, 2014; Persson and Scaraggi, 2014; Persson, 2002), but no simple theories exist which permit to estimate the JKR regime, including negative loads and pull-off, except for Fuller and Tabor (1975) asperity theory, which however has been questioned by Pastewka and Robbins (2014), and certainly contains many strong approximations inherent in the asperity model. In the DMT regime, a very simple solution was given by Ciavarella (2017a) with a "bearing-area" model, which turned out to give very reasonable fit of the Pastewka and Robbins (2014) pull-off data, whereas some discrepancy was remarked about the area-slope "stickiness" criterion with their own pull-off data. Persson (2002) is aimed at the JKR regime, seems perfectly reversible, is quite complex and anyway it is probably not valid in unloading as shown in the plots in Persson and Scaraggi (2014) which only show the positive load regime – and also as explained in details by Carbone et al. (2015) who have constructed for 1D profiles, loading and unloading curves and PSD (Power Spectrum Density) of the deformed profile closely follows a power law predicted by Persson's theory, but not on unloading. In particular, they explain why Persson's theory is not adequate for adhesion. Moving to the DMT approximation of Persson and Scaraggi (2014), the contact is assumed to be split into "repulsive" contact areas and "attractive" contact areas, and no effect of tensile tractions occurs so there is a simple convolution of separation of the repulsive solution with the force-separation law — however, the DMT approximation leads to large errors even in the simple case of a sphere or a cylinder (Ciavarella, 2017b), and it is unclear what happens for rough contacts where many further approximations are made. In any case, the solution remains numerical and not simple in this case either.

All these models are purely theoretical or numerical. Experimental studies typically rely on spherical geometry like Fuller and Tabor (1975).

JKR (1971) originally derived an energetic method which could serve as an approximate solution to a much more general contact case, not just including halfspace geometries but really anything for which we know the adhesionless solution. We shall therefore generalize the JKR model to arbitrary contact geometry, in an approximate sense, in the present paper.

2. The model

We need to consider the total potential energy of the system comprising elastic strain energy, surface energy, and [when load P is prescribed] potential energy of the applied force.

The elastic strain energy can be determined by devising the original JKR loading scenario leading to the required final state and calculating the work done during loading. Such scenario is suggested by the superposition in comprising the two steps (see Fig. 1)

- (i) "repulsive" loading without adhesive forces until the contact area is a given value (this is the load path $\overline{OA}$ as in the original JKR paper), followed by