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A 2-D model for friction of complex anisotropic surfaces

Gianluca Costagliola[a], Federico Bosia[a], Nicola M. Pugno[b,c,d,*]

[a] Department of Physics and Nanostructured Interfaces and Surfaces Centre, University of Torino, Via Pietro Giuria 1, Torino, 10125, Italy
[b] Laboratory of Bio-Inspired & Graphene Nanomechanics, Department of Civil, Environmental and Mechanical Engineering, University of Trento, Via Mesiano, 77, Trento 38123, Italy
[c] School of Engineering and Materials Science, Queen Mary University of London, Mile End Road, London E1 4NS, UK
[d] Ket Labs, Edoardo Amaldi Foundation, Italian Space Agency, Via del Politecnico snc, Rome 00133, Italy

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ABSTRACT

The friction force observed at macroscale is the result of interactions at various lower length scales that are difficult to model in a combined manner. For this reason, simplified approaches are required, depending on the specific aspect to be investigated. In particular, the dimensionality of the system is often reduced, especially in models designed to provide a qualitative description of frictional properties of elastic materials, e.g. the spring-block model. In this paper, we implement for the first time a two dimensional extension of the spring-block model, applying it to structured surfaces and investigating by means of numerical simulations the frictional behaviour of a surface in the presence of features like cavities, pillars or complex anisotropic structures. We show how friction can be effectively tuned by appropriate design of such surface features.

1. Introduction

The frictional behavior of macroscopic bodies arises from various types of interactions occurring at different length scales between contact surfaces in relative motion. While it is clear that their ultimate origin lies in inter-atomic forces, it is difficult to scale these up to the macroscopic level and to include other aspects such as dependence on surface roughness, elasticity or plasticity, wear and specific surface structures (Nosonovsky and Bhushan, 2007; Persson, 2000). Moreover, the dependence on “external parameters”, e.g. relative sliding velocity of the surfaces and normal pressure, is neglected in approximate models such as the fundamental Amontons–Coulomb law, and violations of the latter have been observed (Deng et al., 2012; Katano et al., 2014).

For these reasons, simplified models are required in theoretical studies and numerical simulations, and friction problems can be addressed in different ways depending on the specific aspects under consideration. In order to improve theoretical knowledge of friction, or to design practical applications, it is not necessary to simulate all phenomena simultaneously, and a reductionist approach can be useful to investigate individual issues. Thus, despite the improvement in computational tools, in most cases it is still preferable to develop simplified models to describe specific aspects, aiming to provide qualitative understanding of the fundamental physical mechanisms involved.

One of the most used approaches to deal with friction of elastic bodies consists in the discretization of a material in springs and masses, as done e.g. in the Frenkel–Kontorova model (Braun and Kivshar, 2004), or the Burridge–Knopoff model

* Corresponding author.
E-mail addresses: gcostagl@unito.it (G. Costagliola), fbosia@unito.it (F. Bosia), nicola.pugno@unitn.it (N.M. Pugno).

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(Burridge and Knopoff, 1967), the latter also known as the spring-block model. For simplicity, these models are often formulated in one dimension along the sliding direction, in various versions depending on the specific application. In recent years, interesting results have been obtained with these models, explaining experimental observations (Amundsen et al., 2012; Bouchbinder et al., 2011; Maegawa et al., 2010; Rubinstein et al., 2004; Scheibert and Dysthe, 2010; Trømborg et al., 2011). The extension to two dimensions is the straightforward improvement to better describe experimental results and to correctly reproduce phenomena occurring in two dimensions. This has already been done for some systems, like the Frenkel–Kontorova model (Mandelli et al., 2015; Norell et al., 2016) and the spring-block model applied to geology (Andersen, 1994; Brown et al., 1991; Giacco et al., 2014; Mori and Kawamura, 2008a; Mori and Kawamura, 2008b; Olami et al., 1992), but much work remains to be done to describe the friction of complex and structured surfaces.

The interest of this study lies not only in the numerical modeling of friction in itself, but also in the practical aspects that can be addressed: there are many studies relative to bio-inspired materials (Baum et al., 2014; Li et al., 2016; Murarash et al., 2011; Yurdumakan et al., 2005) or biological materials (Autumn et al., 2000; Labonte et al., 2014; Stempfle and Brendle, 2006; Stempfle et al., 2009; Varenberg et al., 2010) that involve non-trivial geometries that cannot be reduced to one-dimensional structures in order to be correctly modeled.

The one dimensional spring-block model was originally introduced to study earthquakes (Carlson and Langer, 1989; Carlson et al., 1994; Xia et al., 2005) and has also been used to investigate many aspects of dry friction of elastic materials (Amundsen et al., 2015; Braun et al., 2009; Capozza and Pugno, 2015; Capozza et al., 2011; Capozza and Urbakh, 2012; Pugno et al., 2013; Trømborg et al., 2015). In Costagliola et al. (2016), we have extensively investigated the general behavior of the model and the effects of local patterning (regular and hierarchical) on the macroscopic friction coefficients, and in Costagliola et al. (2017a) we have extended the study to composite surfaces, i.e. surfaces with varying material stiffness and roughness; finally in Costagliola et al. (2017b) we have introduced the multiscale extension of the model to study the statistical effects of surface roughness across length scales.

In this paper, we propose a 2-D extension of the spring-block model to describe the frictional behavior of an elastic material sliding on a rigid substrate. Our main aim is to compare the results with those obtained in the one-dimensional case and to extend the study to more complex surface structures, e.g. isotropic or anisotropic arrangements of cavities or pillars, simulating those found in biological materials. The two-dimensional spring-block model allows to consider a more realistic cases and captures a variety of behaviors that can be interesting for practical applications.

The paper is organized as follows: in Section 2, we present the model; in Section 3.1, we discuss the main differences with the one-dimensional case and we explore the role of the parameters without surface structures, highlighting the phenomenology of the model; in Section 3.2, we present the results for standard 1-D and 2-D surface structures like grooves and cavities; in Sections 3.3 and 3.4, we consider more complex cases of anisotropic surface patterning; finally, in Section 4, conclusions and future developments are discussed.

2. Model

The equation of motion for an isotropic linear elastic body driven by a slider on an infinitely rigid plane with damping and friction can be written as: $\rho\ddot{u} = \mu\nabla^2 u + (\lambda+\mu)\nabla(\nabla\cdot u) - \gamma\rho\dot{u}$, where u is the displacement vector, ρ is the mass density, γ is the damping frequency, λ, μ are the Lamé constants. The following boundary conditions must be imposed: the top surface of the body is driven at constant velocity v, the bottom surface is subjected to a spatially variable local friction force, which we discuss below, representing the surface interactions between the elastic body and the rigid plane, while free boundary conditions are set on the remaining sides.

In order to simulate this system, we extend the spring-block model to the two-dimensional case: the contact surface is discretized into elements of mass m, each connected by springs to the eight first neighbors and arranged in a regular square mesh (Fig. 1) with N_x contact points along the x-axis and N_y contact points along the y-axis. Hence, the total number of blocks is $N_b \equiv N_x N_y$. The distances on the axis between the blocks are, respectively, l_x and l_y. The mesh adopted in previous studies of the 2-D spring-block model, e.g. Olami et al. (1992) and Giacco et al. (2014), does not include diagonal springs, but we add them to account for the Poisson effect (our mesh in similar to that used in Trømborg et al. (2011)).

In order to obtain the equivalence of this spring-mass system with a homogeneous elastic material of Young's modulus E, the Poisson's ratio must be fixed to $\nu = 1/3$ (Absi and Prager, 1975), which corresponds to the plane stress case, $l_x = l_y \equiv l$ and $K_{int} = 3/8El_z$, where l_z is the thickness of the 2-D layer and K_{int} is the stiffness of the springs connecting the four nearest neighbors of each block, i.e. those aligned with the axis. The stiffness of the diagonal springs must be $K_{int}/2$. Hence, the internal elastic force on the block i exerted by the neighbor j is $\mathbf{F}_{int}^{(ij)} = k_{ij}(r_{ij} - l_{ij})(\mathbf{r}_j - \mathbf{r}_i)/r_{ij}$, where $\mathbf{r}_i$, $\mathbf{r}_j$ are the position vectors of the two blocks, r_{ij} is the modulus of their distance, l_{ij} is the modulus of their rest distance and k_{ij} is the stiffness of the spring connecting them.

All the blocks are connected, through springs of stiffness K_s, to the slider that is moving at constant velocity v in the x direction, i.e. the slider vector velocity is $\mathbf{v} = (v, 0)$. Given the initial rest position $\mathbf{r}_i^0$ of block i, the shear force is $\mathbf{F}_s^{(i)} = K_s(\mathbf{v}t + \mathbf{r}_i^0 - \mathbf{r}_i)$. We define the total driving force on i as $\mathbf{F}_{mot}^{(i)} = \sum_j \mathbf{F}_{int}^{(ij)} + \mathbf{F}_s^{(i)}$. The stiffness K_s can be related to the macroscopic shear modulus $G = 3/8E$, since all the shear springs are attached in parallel, so that by simple calculations we obtain $K_s = K_{int}l^2/l_z^2$. In the following, for simplicity we fix $l_z = l$. This formulation, commonly used in spring-block models, neglects the long-range interactions that may arise from wave propagation through the bulk (Elbanna 2011; Hulikal et al.,