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A general multiscale framework for the emergent effective elastodynamics of metamaterials

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ABSTRACT

This paper presents a general multiscale framework towards the computation of the emergent effective elastodynamics of heterogeneous materials, to be applied for the analysis of acoustic metamaterials and phononic crystals. The generality of the framework is exemplified by two key characteristics. First, the underlying formalism relies on the Floquet–Bloch theorem to derive a robust definition of scales and scale separation. Second, unlike most homogenization approaches that rely on a classical volume average, a generalized homogenization operator is defined with respect to a family of particular projection functions. This yields a generalized macro-scale continuum, instead of the classical Cauchy continuum. This enables (in a micromorphic sense) to homogenize the rich dispersive behavior resulting from both Bragg scattering and local resonance. For an arbitrary unit cell, the homogenization projection functions are constructed using the Floquet–Bloch eigenvectors obtained in the desired frequency regime at select high symmetry points, which effectively resolves the emergent phenomena dominating that regime. Furthermore, a generalized Hill–Mandel condition is proposed that ensures power consistency between the homogenized and full-scale model. A high-order spatio-temporal gradient expansion is used to localize the multiscale problem leading to a series of recursive unit cell problems giving the appropriate micro-mechanical corrections. The developed multiscale method is validated against standard numerical Bloch analysis of the dispersion spectra of example unit cells encompassing multiple high-order branches generated by local resonance and/or Bragg scattering.

1. Introduction

Acoustic metamaterials and phononic crystals are specially designed composites enabling extraordinary manipulation of mechanical waves, such as band-stop filtering, redirection, channeling, multiplexing etc., which is impossible with conventional materials (Deymier, 2013; Hussein et al., 2014). These properties are a result of two distinct scattering phenomena occurring separately or jointly, i.e. Bragg scattering and local resonance. The emergent wave resulting from both Bragg scattering and local resonance is coherent and is associated with a well defined dispersion spectrum. The most striking feature

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of such a spectrum is the appearance of bandgaps, which are the frequency ranges where no real (i.e. propagating) wave solution exists. The exotic properties of these metamaterials are mostly a consequence of these physical phenomena.

This paper presents a multiscale modeling framework for acoustic metamaterials and phononic crystals restricted to the linear elastic regime without dissipation. The underlying motivation consists in the development of efficient computational techniques for transient structural dynamics analyses of acoustic metamaterials and phononic crystals with arbitrary rich dispersive behavior in reasonably broad frequency regimes. Although the multiscale analysis techniques developed thus far (Hui and Oskay, 2014; Liu and Reina, 2017; Pham et al., 2013; Sridhar et al., 2016) have demonstrated an advantage for solving large scale problems compared to the direct numerical simulations of the fully resolved models, the homogenization theories underlying these methods are limited to certain special microstructures within a narrow frequency range of validity (typically in the low frequency regime). These limitations are a consequence of the definition of scales and the underlying scale separation justifying these approaches. This becomes clear through the adopted homogenization formulation, i.e.

- The kinematic and dynamic assumptions made on the micro-scale fluctuations resulting from the material heterogeneities.
- The formulation of the homogenization (/smoothening/regularization) operator that is used to project (upscale) the micro-scale quantities onto the macro-scale.

In the low frequency, long-wavelength regime (where no dispersion is observed), a quasi-static assumption on the micro-scale fluctuations subjected to periodic boundary conditions is justified and the homogenization operator is taken to be a uniform volume average over the unit cell as required by two-scale convergence (Allaire, 1992). The resulting homogenized model retains the Cauchy continuum form with local constitutive response. Extensions to this model have mostly been focused on relaxing the assumptions on the micro-scale fluctuations, while little attention has been paid towards generalizing the homogenization operation beyond the uniform volume average.

Higher order spatial and/or temporal asymptotic corrections to the quasi-static, periodic micro-fluctuations have been proposed (Andrianov et al., 2008; Bacigalupo and Gambarotta, 2014; Chen and Fish, 2000; Hu and Oskay, 2017; Hui and Oskay, 2014) that extend the classical theory to shorter wavelength regimes. The resulting effective constitutive equations are (weakly) nonlocal, containing higher order spatial and/or temporal derivatives of the displacements. Such models are capable of capturing the dispersion of the initial (also called acoustic) branches and sometimes, the second-order (also called optical) branches. An important step towards developing a general homogenization framework (for geometrically periodic microstructures) has been investigated extensively in the works of Willis (1997; 2009; 2011; 2012) and further developed by other authors (Nassar et al., 2015; 2016; Srivastava and Nemat-Nasser, 2012; 2014) that fully relaxes any assumptions on micro-scale fluctuations and extends the range of possible macro-scale wavelengths to the entire Brillouin zone. Here, the general solution for the micro-scale fluctuations is provided by the Floquet–Bloch theorem (Bensoussan et al., 1978; Gazalet et al., 2013). The resulting effective constitutive model is strongly nonlocal in both space and time. Furthermore, an additional coupling between the homogenized stress and velocity, and momentum and strain has also been demonstrated. The potential of this approach in terms of capturing Bragg scattering branches in 3D composites has been shown in Srivastava and Nemat-Nasser (2012). However, despite its generality even these approaches are theoretically limited to only a few low-order branches (Srivastava and Nemat-Nasser, 2014). The reason for this limitation was identified in Nassar et al. (2015), where the use of the uniform volume averaging operator was shown to be suboptimal at frequencies beyond the low-order branches.

At higher frequencies, exotic wave modes emerge especially for 2D and 3D unit cell geometries, which can transport energy even at vanishing average displacement, e.g. rotational/Cosserat waves (Deymier et al., 2014). Indeed, in such cases, it is necessary to resort to a micromorphic macro-scale continuum description of the type hypothesized by Eringen (1999) and Mindlin (1964), Germain (1973) and Madeo et al. (2015), instead of the classical Cauchy continuum. Furthermore, micro-inertial effects dominate at these frequencies where adjoining material domains within the unit cell start to vibrate out of (harmonic) phase with respect to each other. In this case, it is necessary to recover the phase information which is lost through uniform volume averaging. It is therefore necessary to generalize the formulation of the homogenization operation by introducing an appropriate weighted average that properly captures the emergent phenomena upon projection at the macro-scale.

The formulation of this weighted projection is well understood for local resonance based metamaterials e.g. Liu et al. (2000), where the inclusion vibrates out of phase with respect to the matrix beyond the local resonance frequencies. The volume average is performed solely over the matrix whereas the dynamics of the inclusion are seperately modeled, normally on the subspace of local resonance eigenmodes of the inclusion (Auriault and Boutin, 2012; Milton and Willis, 2007; Smyshlyaev, 2009; Willis, 2009). Computational homogenization techniques for exclusively modeling local resonance acoustic metamaterials have successfully demonstrated this (Pham et al., 2013; Sridhar et al., 2016). The formulation of the homogenization operator for a general microstructure that can also exhibit Bragg scattering is less straightforward. Relevant work in this direction include (Boutin et al., 2014; Craster et al., 2010), where the Floquet-Bloch eigenvector of the unit cell, obtained via dispersion analysis, at a given point on the Brillouin zone was proposed as the projection function for the homogenization operator. The modified homogenization theory allowed capturing exactly, the dispersion spectrum in the proximity of the eigenfrequency of the Floquet-Bloch mode. Taking this concept further, the homogenization operation was enriched in Nassar et al. (2016) using a family of projection functions in addition to the uniform one, in order to capture multiple emergent phenomena and the corresponding dispersion branches. The method was validated on a simple