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The effective compliance of spatially evolving planar wing-cracks

R.S. Ayyagari[a,c,1,*], N.P. Daphalapurkar[b], K.T. Ramesh[a,b]

[a] *Hopkins Extreme Materials Institute, Johns Hopkins University, Baltimore, MD 21218, USA*
[b] *Department of Mechanical Engineering, Johns Hopkins University, Baltimore, MD 21218, USA*
[c] *Mechanical Engineering, Indian Institute of Technology, Gandhinagar, Gujarat 382355, India*

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ABSTRACT

We present an analytic closed form solution for anisotropic change in compliance due to the spatial evolution of planar wing-cracks in a material subjected to largely compressive loading. A fully three-dimensional anisotropic compliance tensor is defined and evaluated considering the wing-crack mechanism, using a mixed-approach based on kinematic and energetic arguments to derive the coefficients in incremental compliance. Material, kinematic and kinetic parametric influences on the increments in compliance are studied in order to understand their physical implications on material failure. Model verification is carried out through comparisons to experimental uniaxial compression results to showcase the predictive capabilities of the current study.

1. Introduction

Ceramics, masonry, sandstone and other brittle materials demonstrate a nonlinear failure response to high-rate compression. This includes degradation in elastic constants, development of load-induced anisotropy and bulking. This complex behavior is attributed to the growth of wing-cracks activated from pre-existing microdefects inherent in these materials. Randomly distributed flaws, under compressive loads, exhibit sliding on flaw surfaces resulting in nucleation and growth of tension cracks, commonly referred to as wing-cracks (Nemat-Nasser and Obata, 1988; Paliwal and Ramesh, 2008). Such materials are typically weak in tension, compared to their strength in compression, and wing-cracks grow under a local tensile stress state, despite the overall compressive load. Evolving wing cracks align in the direction of maximum compressive load when fully developed. These wing-cracks further interact and coalesce, contributing to inelastic strain and subsequently resulting in global failure (Horii and Nemat-Nasser, 1983; 1985; 1986; Nemat-Nasser and Horii, 1982). The objective of this paper is to present analytic closed-form expressions for the effective anisotropic compliance of a brittle material permeated by spatially evolving planar wing cracks.

Studies on failure in brittle materials with wing-cracks as the dominant mechanism have contributed to understanding (a) the influence of damage on effective material properties (Horii and Nemat-Nasser, 1983; Kachanov and Sevostianov, 2005; Lubarda and Krajcinovic, 1994; Nemat-Nasser and Obata, 1988; Ravichandran and Subhash, 1995) and (b) the nucleation, growth and coalescence of cracks under quasi-static and dynamic regimes (Basista and Gross, 1998; Deshpande and Evans, 2008; Nemat-Nasser and Obata, 1988; Paliwal and Ramesh, 2008; Ravichandran and Subhash, 1995; Shao and Rudnicki, 2000; Zhou and Yang, 2007). Numerous investigations estimate the effective elastic properties of bodies with randomly

* Corresponding author.
E-mail addresses: rayyaga2@jhu.edu, ravi.ayyagari@iitgn.ac.in (R.S. Ayyagari), nitin@jhu.edu (N.P. Daphalapurkar), ramesh@jhu.edu (K.T. Ramesh).
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distributed cracks under tensile loading. The majority of these studies incorporate one of three approaches to explicitly define the degraded modulus through closed-form expressions: (a) non-interacting crack theory (Kachanov, 1992; 1982), (b) differential schemes (Benveniste, 1986) or (c) self-consistent approximations (Budiansky and O'Connell, 1976). Flat circular crack geometries have been an underlying assumption in a number of these studies, with a seminal extension to elliptical cracks by Budiansky and O'Connell (1976). Several investigations have concluded that multiple irregular flat cracks can be replaced by an equivalent set of circular cracks, if geometric irregularities are random (Grechka and Kachanov, 2006a; Kachanov and Sevostianov, 2005; Sevostianov and Kachanov, 2002). Further, the effect of intersecting planar cracks on the overall effective properties was found to be minimal by Grechka and Kachanov (2006b).

However, under compression loading, contact of discontinuous crack surfaces and the ensuing frictional sliding make it more difficult to estimate the effective elastic properties. Rate constitutive relations for loading and unloading assuming a dilute distribution of pre-existing flaws (specifically for wing-cracks in a two dimensional setting) have been extensively studied by Nemat-Nasser and Obata (1988). Their model defines inelastic strain as a function of sliding displacement and derives effective material properties based on stress intensities and kinematic arguments. Basista and Gross (1998) developed a two-dimensional damage model that is based on sliding cracks as the dissipative mechanism for inelastic deformation. This model utilizes Rice's internal variable thermodynamic framework, and in the long wing-crack regime reduces to the relations developed by Nemat Nasser and Obata that are obtained from kinematic analyses.

Interest in obtaining the effective elastic properties stems from the need to develop reliable and robust micromechanics informed damage models that can handle generalized stress states. Paliwal and Ramesh (2008) developed a two-dimensional constitutive model that is micromechanically informed and based on the sliding wing-crack mechanism. An isotropic damage parameter was defined that evolves through a self-consistent formalism accounting for flaw interactions. Their model is capable of incorporating the pre-existing flaw statistics to determine the constitutive response. Stresses were estimated by assuming effective properties evaluated for traction free elliptic cracks (Budiansky and O'Connell, 1976) as a function of the evolving damage parameter. Recent developments have seen extensions of this isotropic damage approach to three dimensions (Hu et al., 2015; Tonge et al., 2013). However, extensive experimental investigations (Farbaniec et al., 2015; Hogan et al., 2015) have established that anisotropic damage often dominates the response of advanced ceramics. Anisotropic dynamic damage was introduced by Hu et al. (2015), through an adhoc tensorial description that also implicitly incorporates flaw statistics. Damage evolution in their model is dictated by dynamic crack growth kinetics and utilizes a modified form of Kachanov's effective compliance relation (Grechka and Kachanov, 2006a) to update the stress measures. This approach has been effective for simple stress paths.

Several other works from literature focus on estimating the anisotropic damage based on micro-mechanical approaches (Lee and Ju, 1991; Pensée et al., 2002; Zhu et al., 2008). Lee and Ju (1991) present a micro mechanics based anisotropic damage model for brittle solids under compressive triaxial loads, and conclude that the overall compliances are non-symmetric and anisotropic. However, the compliances were numerically estimated based on penny-shaped micro-crack opening displacements. Zhu et al. (2008) incorporate a variation of the self-consistent method developed by Castañeda and Willis (1995) that accounts for the spatial distribution of inclusions to present an anisotropic damage model under compressive loads. They investigated the role of spatial distribution of micro-cracks and crack-crack interactions on the damage response by comparing three homogenization schemes that lead to effective compliance measures. Homogenized measures of stiffness and compliance have also been derived for micro-cracked materials by Pensée et al. (2002) using continuum damage mechanics approach as well as micro-mechanical approach. Based on their study, they conclude that the micro-mechanical models appear to be particularly appropriate for describing damage under tensile loads. Recent advances have seen coupling of micro-mechanics and continuum damage mechanics principles to investigate crack growth both under tensile and compressive loads simultaneously (Jin and Arson, 2017a).

The full potential of these micromechanics based self-consistent dynamic compressive damage models has not been realized because they estimate the effective elastic properties through damage-compliance relationships that were developed for cracked solids under tensile loading. Such an approach overestimates the compliance in the loading direction and underestimates the dilatation at a specified damage state (Liu, 2015). These approaches provide an encouraging baseline for development of a closed-form solution for the effective elastic properties of a solid accumulating damage due to growth of a statistical distribution of wing-cracks.

Here we determine the effective elastic properties for a generalized tensorial brittle damage model under compressive states. Derivations of the increment in compliance due to the presence of a single planar wing-crack are handled in a three-dimensional setting through a mixed approach, utilizing crack release energies (Budiansky and O'Connell, 1976) on the one hand and crack opening displacements (Nemat-Nasser and Obata, 1988) on the other. Crack interactions are accounted for through the self-consistent approach. The pristine parent material (matrix) is assumed to be isotropically elastic. Anisotropy develops due to preferential crack growth and is a natural consequence of the analytical framework. The analytical framework gives explicit expressions for the effective compliances, and has advantages over all previous approaches because of the capability to simultaneously handle generalized stress states, flaw statistics and self-consistent crack interactions.

The article is organized as follows. First, the kinematics of a wing-cracked solid is used to arrive at an anisotropic damage measure described using a second order tensor. The damage definition is further used to establish the associated inelastic strain and damage compliance. Constants within the inelastic strain and the damage-compliance are determined based on kinematic and energetic consistency. The effective compliance of the system is then derived as a function of wing-crack length and a three-dimensional generalized stress state. Finally, the generalized relation for inelastic strain is demonstrated