Journal of the Mechanics and Physics of Solids 110 (2018) 155–172

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

The elasto-plastic behaviour of three-dimensional stochastic fibre networks with cross-linkers

Y.H. Ma[a], H.X. Zhu[a,*], B. Su[b], G.K. Hu[c], R. Perks[a]

[a] School of Engineering, Cardiff University, Cardiff CF243AA, UK
[b] School of Oral and Dental Sciences, University of Bristol, Bristol BS12LY, UK
[c] School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China

A R T I C L E I N F O

Article history:
Received 21 August 2017
Accepted 25 September 2017
Available online 29 September 2017

Keywords:
Fibre network materials
Yield surface
Relative density
Transverse isotropy
Modelling
Multiaxial

A B S T R A C T

Fibre network materials constitute a class of highly porous materials with low density, promising for functional and structural applications; however, very limited research has been conducted, especially on simulation and analytical models. In this paper, a continuum mechanics-based three-dimensional periodic beam-network model has been constructed to describe the stochastic fibre network materials. In this model, the density of the cross-linkers is directly related to the relative density of the fibre network materials, and the cross-linkers are represented by equivalent beam elements. The objective of this work was to delineate the elasto-plastic behaviour of the stochastic fibre network materials. Characteristic stress and strain derived from the total strain energy density have been adopted to reveal the yielding behaviour of the fibre networks. The results indicate that the stochastic fibre network materials are transversely isotropic. The in-plane stiffness and strength are much larger than those in the out-of-plane direction. For the fibre network materials with a small relative density, the relationship between the uniaxial yield strength and the relative density is a quadratic function in the x direction and is a cubic function in the z direction, which agree well with our dimensional analysis and are consistent with the relevant experimental results in literature. The yield surface depends strongly on the relative density and the connection between fibres.

1. Introduction

Cellular materials have been of great interest to engineers and scientists due to their attractive mechanical and physical properties and their wide applications. Foams and honeycombs, which are categorized as cellular materials, have been extensively studied (Banhart, 2001; Chen et al., 1999; Gibson and Ashby, 1997; Silva and Gibson, 1997; Zhu et al., 2000, 1997). As Gibson and Ashby (1997) described, the four major engineering applications of cellular materials are thermal insulation, packaging, structural use and buoyancy attributed to their low thermal conductivity, high compressive strength, high stiffness and low density. With the same attractive properties as other cellular materials, porous fibrous materials are less researched and understood because of their more complex geometry. Some techniques have been developed to pro-

* Corresponding author.
E-mail address: zhuh3@cf.ac.uk (H.X. Zhu).

https://doi.org/10.1016/j.jmps.2017.09.014
0022-5096/

duce fibrous materials, for instance, fibre pull-out techniques (Xi et al., 2011) for porous metal fibre sintered sheets (MFSSs) (Jin et al., 2013) which can be used in sandwich panel cores, and electrospinning techniques (Huang et al., 2003; Zhang et al., 2008) for polymer fibre scaffold used in drug delivery and tissue engineering (Sill and von Recum, 2008). In the architecture of honeycombs and foams, cells are distinct and a single idealized unit cell can be used to represent the microstructural features, e.g., 2D honeycombs in which a representative element can be a single triangular, square or hexagonal cell (Gibson and Ashby, 1997; Zhu et al., 2012). Many analytic and simulation models have been developed to investigate the macro-mechanical behaviour of honeycombs and foams (Chen et al., 1999; Redenbach, 2009; Silva and Gibson, 1997; Youssef et al., 2005; Zhu et al., 2001, 2000, 1997). In contrast, the architecture is much more complicated for fibrous materials, for instance, felt, as it is very difficult to define an individual cell in the architecture. Paper is a typical stochastic fibrous material which is, to some extent, the most familiar one to most of us. Porous metal fibre sintered sheets (MFSSs) are metal fibrous materials with good balance between density and strength, and have a great potential for functional and structural applications. MFSSs are produced by overlapping the randomly distributed fibres by air-laid web-forming technology before compressing and sintering into three-dimensional materials with a fibre network (Xi et al., 2011). Experiments (Zhao et al., 2013) show that the MFSSs are transversely isotropic and that the in-plane stiffness and strength are much higher than those in the out-of-plane direction, which is different from foams or honeycombs. Connectivity in the fibre network materials plays a significant role in determining the mechanical properties. It was found that the in-plane deformation of porous materials with high nodal connectivity is stretching dominated. In contrast, the deformation mode of low density two-dimensional hexagonal honeycomb is a combination of cell wall stretching and bending when the nodal connectivity is low (Symons and Fleck, 2008). Jin et al. (2013) developed a two-dimensional random micromechanics beam model to investigate the in-plane elasto-plastic behaviour of MFSSs (Jin et al., 2013). In their model, all the fibres are completely overlapped with each other to form a two-dimensional stochastic fibre network, which leads to very strong bonding connections and high nodal connectivity between fibres. Xi et al. (2011) proposed that the mechanical properties of metal fibre porous materials are highly dependent upon the fibre-fibre joints and the number of metallurgy nodes. The mechanical properties are enhanced with increasing sintering contact points per unit volume and the bonding intensity. Some analytical models have been developed based on the assumption of affine deformation of the fibre network (Clyne et al., 2005; Markaki and Clyne, 2005; Picu, 2011; Tsarouchas and Markaki, 2011).

The relative density can significantly affect the strength and stiffness of porous materials (Gibson and Ashby, 1997; Chen et al., 1999; Jin et al., 2013; Won et al., 2013; Zhu et al., 2001, 2000, 1997); however, it is difficult to control the relative density in the manufacturing process. Finite element method (FEM) which was originally developed for solving solid mechanics problems offers a means to probe the mechanical properties of intricate stochastic fibrous materials by controlling the relative density and other key parameters in the model. In addition, optimized design of complex porous materials can be realized by finite element method. In Finite Element (FE) modelling, beams are mostly utilized to represent fibres and the way to treat the connections between the beams/fibres is crucial. A comprehensive study on the modelling of stochastic fibrous materials by mathematical treatment, for instance, the probability and distribution, can be found in reference (Sampson, 2008); however, the connection between fibres was not taken into consideration. Sastry and co-workers (Sastry et al., 2001; Wang et al., 2000; Wang and Sastry, 2000) proposed a technique for modelling fibre–fibre joints in which connection realized by a torsion spring can be regarded as flexible; however, the mechanical properties of fibrous networks with flexible bonding were not given. It is suggested that the connectivity between fibres cannot be adequately described by single connection points in the beam modelling (Wang and Sastry, 2000).

In this study, we have incorporated the density of intersections (or connections) into our model. This is important because the density of intersections is directly related to the relative density in stochastic fibrous materials. In our fibre network model, there is no fibre entanglement which is commonly found in woven fabrics (Grishanov et al., 2012). It is generally recognised that the macroscopic stresses and strains can be determined by the microscopic stresses and strains over a representative volume element (RVE). Hill (1963) proposed that a representative 'cell unit' (or a representative volume element) (RVE) can be a full-scale model to significantly reduce the computation complexity.

Traditionally the yielding of a material is defined according to the von Mises equivalent stress versus strain curve; however, it cannot be used to describe the yielding of a porous material when it is subjected to hydrostatic loading as the von Mises criterion is only based on the distortional part of the stored strain energy density. Some researchers have put forward the characteristic stress and strain which combine the hydrostatic energy density and deviatoric energy density to probe the yielding of two-dimensional isotropic honeycombs (Chen et al., 1999; Chen and Lu, 2000), two-dimensional anisotropic cellular materials (Alkhader and Vural, 2009) and three-dimensional transversely isotropic foams (Ayyagari and Vural, 2015; Zhao et al., 2013). The deduction of characteristic stress and strain is based on the total stored strain energy density and is different from those phenomenological yield criterions, for instance, a shape parameter needs to be given to describe the mean-effective stress (Deshpande and Fleck, 2000).

It has been suggested that there exists a significant difference between different types of cellular materials, such as foams and honeycombs, and fibre network materials (Markaki and Clyne, 2003; Qiao et al., 2009; Zhao et al., 2013). It is crucial to build quantitative mechanical models to reveal the mechanical behaviour of stochastic fibre network materials. Due to the computation limitation, we have used a RVE to represent a whole fibre network material. The RVE thus must be periodic. It has been shown that periodic boundary conditions are more suitable than either mixed boundary conditions or prescribed displacement boundary conditions (Chen et al., 1999; Zhu et al., 2000, 2001). To begin with, a three-dimensional periodic beam network model for stochastic fibre network materials has been constructed. We have introduced a novel method to