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Dependence of rock properties on the Lode angle: Experimental data, constitutive model, and bifurcation analysis

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ABSTRACT

The overwhelming majority of experimental tests on rocks have only been conducted for a single value of the Lode angle θ corresponding to the axisymmetric compression (AC). There are now sufficiently extensive data sets from both AC and axisymmetric extension (AE) tests (corresponding to two extreme θ values) for two materials (synthetic rock analog GRAM1 and Solnhofen Limestone). These data cover a wide range of the confining pressure (from brittle faulting to ductile flow). Very recently the data from true 3-D tests (for different θ) also covering both brittle and ductile fields were published for Castlegate and Bentheim Sandstone as well. The results from all these tests summarized and processed in this paper constitute a solid basis which allows general conclusions to be drawn about the dependence of rock behavior on θ . In all cases, the yield/failure envelopes were shown to be θ -dependent so that the material strength at low mean stress σ is smaller under AE than under AC, while at high σ , it is the opposite. The brittle-ductile transition under AE occurs at $\sigma \sim 1.5$ times greater than under AC, meaning that under AE the material is more prone to fracture development. The angle between the most compressive stress and the forming deformation localization bands is systematically higher for AE than for AC for the same σ . Based on these data we formulate a new three-invariant constitutive model with convex and concave yield functions (YFs) which is used for the bifurcation analysis. The results of this analysis agree with the experimental data (for both YFs) and reveal that the θ -dependence of rock properties encourages the strain localization. The major factors defining this dependence are the θ -dependence of the YFs but also of the dilatancy factor which is greater for AE than for AC. The theoretical results show that the failure (deformation band) plane can deviate from the intermediate stress direction and can become parallel to the maximum compressive stress at high σ for the concave YF. © 2016 Elsevier Ltd. All rights reserved.

1. Introduction

It has long been known that the strength of geomaterials under compression $\bar{\tau}_c^{pk}$ can be considerably higher than that under extension $\bar{\tau}_{ex}^{pk}$ for the same mean stress σ (Mogi, 1967, 1971; Chang and Haimson, 2000; Haimson and Chang, 2000; Haimson and Rudnicki, 2010; Haimson, 2011; Lee and Haimson, 2011), where $\bar{\tau}$ is the von Mises stress, the superscript "*pk*"

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Nomenclature

Kbulk modulusEYoung's modulus ν Poisson's ratio θ Lode angle σ_i grincipal stresses S_{ij} stress tensor (i, j = 1, 2, 3) σ_i principal stresses S_{ij} stress deviator tensor δ_{ij} Kronecker delta σ_1 maximum compression stress σ_2 intermediate principal stress σ_2 intermediate principal stress σ mean stress σ_{cr} mean stress at the crest of initial yield envelopes J_3 third invariant of stress deviator ψ angle between σ_2 -parallel deformation locali- zation bands (planes) and σ_1 direction $\Delta\psi^{(\theta)} = 0^\circ$ $-\psi(\theta = 60^\circ)$ $\Delta\psi^{(\theta)} = \Delta\psi^{-}$ $\Delta\psi$ for Bentheim and Castlegate sandstones $\Delta\psi^{G}$ $\Delta\psi^{S}$ $\Delta\psi$ for GRAM1 material, and Solnhofen limestone ξ equal to cos 2ψ , Eq. (32). n_i unit normal to deformation localization bands in the principal stress spaceACaxisymmetric compressionAEaxisymmetric extensionYSyield function σ_{bdt} mean stress at brittle-ductile transition for AC and AE, respectively ψ von Mises stress F yield function ϕ_{ex}^{bdt} ψ at brittle-ductile transition for AC and AE, respectively $\bar{\tau}$ von Mises stress F yield function ϕ plastic potential function ϕ_{ex} otential yield functions for AC and AE, respect		shear modulus
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$\begin{array}{lll} \theta & \text{Lode angle} \\ \sigma_{ij} & \text{stress tensor } (i, j = 1, 2, 3) \\ \sigma_i & \text{principal stresses} \\ s_{ij} & \text{stress deviator tensor} \\ \delta_{ij} & \text{Kronecker delta} \\ \sigma_1 & \text{maximum compression stress} \\ \sigma_2 & \text{intermediate principal stress} \\ \sigma & \text{mean stress} \\ \sigma_cr & \text{mean stress at the crest of initial yield} \\ & \text{envelopes} \\ J_3 & \text{third invariant of stress deviator} \\ \psi & \text{angle between } \sigma_2\text{-parallel deformation localization bands (planes) and } \sigma_1 \text{ direction} \\ \psi^* & \text{angle between } \sigma_1\text{-parallel deformation bands} \\ & \text{and } \sigma_2 \text{ direction} \\ \Delta\psi & \psi(\theta = 0^\circ) - \psi(\theta = 60^\circ) \\ \Delta\psi^{B}, \Delta\psi^{C} & \Delta\psi \text{ for Bentheim and Castlegate sandstones} \\ \Delta\psi^{C}, \Delta\psi^{S} & \Delta\psi & \text{ for GRAM1 material, and Solnhofen} \\ & \text{limestone} \\ \xi & \text{equal to cos } 2\psi, \text{ Eq. (32).} \\ n_i & \text{unit normal to deformation localization bands} \\ & \text{in the principal stress space} \\ \text{AC} & \text{axisymmetric compression} \\ \text{AE} & \text{axisymmetric extension} \\ \text{YS} & yield surface} \\ \text{YF} & yield function} \\ \sigma_{bdt}^{bdt} & \text{mean stress at brittle-ductile transition for AC} \\ & \text{and AE, respectively} \\ \psi_c^{bdt}, \psi_{ex}^{bdt} & \psi \text{ at brittle-ductile transition for AC and AE, respectively} \\ \psi_c^{bdt}, \phi_{ex}(\sigma) & \text{initial yield function} \\ \theta & \text{plastic potential function} \\ \bar{\tau}_c(\sigma), \ \bar{\tau}_{ex}(\sigma) & \text{initial yield functions for AC and AE, respectively} \\ \sigma_0 & \text{mean stress at the intersection of } \{\bar{\tau}_c(\sigma), \ \bar{\tau}_{ex}(\sigma) \\ \end{array}$	ν	Poisson's ratio
	θ	Lode angle
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$\begin{split} \delta_{ij} & \text{Kronecker delta} \\ \sigma_1 & \text{maximum compression stress} \\ \sigma_2 & \text{intermediate principal stress} \\ \sigma & \text{mean stress} \\ \sigma_{cr} & \text{mean stress at the crest of initial yield} \\ & \text{envelopes} \\ J_3 & \text{third invariant of stress deviator} \\ \psi & \text{angle between } \sigma_2 \text{-parallel deformation localization bands (planes) and } \sigma_1 \text{ direction} \\ \psi^* & \text{angle between } \sigma_1 \text{-parallel deformation bands} \\ & \text{and } \sigma_2 \text{ direction} \\ \Delta \psi & \psi(\theta = 0^\circ) - \psi(\theta = 60^\circ) \\ \Delta \psi^B, \Delta \psi^C & \Delta \psi \text{ for Bentheim and Castlegate sandstones} \\ \Delta \psi^G, \Delta \psi^S & \Delta \psi \text{ for GRAM1 material, and Solnhofen} \\ & \text{limestone} \\ \xi & \text{equal to cos } 2\psi, \text{Eq. (32).} \\ n_i & \text{unit normal to deformation localization bands} \\ & \text{in the principal stress space} \\ \text{AC} & \text{axisymmetric compression} \\ \text{AE} & \text{axisymmetric extension} \\ \text{YS} & \text{yield surface} \\ \text{YF} & \text{yield function} \\ \sigma_c^{bdt}, \sigma_{ex}^{bdt} & \text{w at brittle-ductile transition for AC} \\ & \text{and AE, respectively} \\ \psi_c^{bdt}, \psi_{ex}^{bdt} & \psi \text{ at brittle-ductile transition for AC} \\ & \text{and AE, respectively} \\ \bar{\tau} & \text{von Mises stress} \\ F & \text{yield function} \\ \phi & \text{plastic potential function} \\ \bar{\tau}_c(\sigma), \ \bar{\tau}_{ex}(\sigma) & \text{initial yield functions for AC and AE, respectively} \\ \sigma_0 & \text{mean stress at the intersection of } \overline{\tau}_c(\sigma), \ \bar{\tau}_{ex}(\sigma) \\ \end{array}$	S _{ij}	stress deviator tensor
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$ \begin{array}{llllllllllllllllllllllllllllllllllll$	AC AE YS	in the principal stress space axisymmetric compression axisymmetric extension yield surface
$ \sigma_c^{bdt}, \sigma_{ex}^{bdt} $ mean stress at brittle-ductile transition for AC and AE, respectively $\psi_c^{bdt}, \psi_{ex}^{bdt} \psi$ at brittle-ductile transition for AC and AE, respectively $\bar{\tau}$ von Mises stress F yield function Φ plastic potential function $\bar{\tau}_c(\sigma), \ \bar{\tau}_{ex}(\sigma)$ initial yield functions for AC and AE, respectively σ_0 mean stress at the intersection of $\bar{\tau}_c(\sigma), \ \bar{\tau}_{ex}(\sigma)$	AC AE YS YF	in the principal stress space axisymmetric compression axisymmetric extension yield surface yield function
and AE, respectively $\psi_c^{bdt}, \psi_{ex}^{bdt} \psi$ at brittle-ductile transition for AC and AE, respectively $\bar{\tau}$ von Mises stress F yield function Φ plastic potential function $\bar{\tau}_c(\sigma), \ \bar{\tau}_{ex}(\sigma)$ initial yield functions for AC and AE, respectively σ_0 mean stress at the intersection of $\bar{\tau}_c(\sigma), \ \bar{\tau}_{ex}(\sigma)$	AC AE YS YF σ^{bdt}	in the principal stress space axisymmetric compression axisymmetric extension yield surface yield function mean stress at brittle-ductile transition
$\begin{array}{ccc} \psi_c^{bat}, \psi_{ex}^{bat} & \psi \text{ at brittle-ductile transition for AC and AE,} \\ & \text{respectively} \\ \bar{\tau} & \text{von Mises stress} \\ F & \text{yield function} \\ \Phi & \text{plastic potential function} \\ \bar{\tau}_c(\sigma), \ \bar{\tau}_{ex}(\sigma) \text{ initial yield functions for AC and AE,} \\ & \text{respectively} \\ \sigma_0 & \text{mean stress at the intersection of } \bar{\tau}_c(\sigma), \ \bar{\tau}_{ex}(\sigma) \end{array}$	AC AE YS YF σ^{bdt} σ^{bdt}_{c} , σ^{bdt}_{ex}	in the principal stress space axisymmetric compression axisymmetric extension yield surface yield function mean stress at brittle-ductile transition mean stress at brittle-ductile transition for AC
respectively $\bar{\tau}$ von Mises stress F yield function Φ plastic potential function $\bar{\tau}_c(\sigma)$, $\bar{\tau}_{ex}(\sigma)$ initial yield functions for AC and AE, respectively σ_0 mean stress at the intersection of $\bar{\tau}_c(\sigma)$, $\bar{\tau}_{ex}(\sigma)$	AC AE YS YF σ^{bdt} σ^{bdt}_{c} , σ^{bdt}_{ex}	in the principal stress space axisymmetric compression axisymmetric extension yield surface yield function mean stress at brittle-ductile transition mean stress at brittle-ductile transition for AC and AE, respectively
$ \vec{\tau} $ von Mises stress F yield function $ \phi $ plastic potential function $ \vec{\tau}_c(\sigma), \ \vec{\tau}_{ex}(\sigma) $ initial yield functions for AC and AE, respectively $ \sigma_0 $ mean stress at the intersection of $ \vec{\tau}_c(\sigma), \ \vec{\tau}_{ex}(\sigma) $	AC AE YS YF σ_c^{bdt} , σ_c^{bdt} , σ_e^{bdt} , ψ_{ex}^{bd}	In the principal stress space axisymmetric compression axisymmetric extension yield surface yield function mean stress at brittle-ductile transition mean stress at brittle-ductile transition for AC and AE, respectively ψ at brittle-ductile transition for AC and AE,
Fyield function	AC AE YS YF σ_c^{bdt} , σ_c^{bdt} , σ_{ex}^{cdt} ψ_c^{bdt} , ψ_{ex}^{bc}	In the principal stress space axisymmetric compression axisymmetric extension yield surface yield function mean stress at brittle-ductile transition mean stress at brittle-ductile transition for AC and AE, respectively ψ at brittle-ductile transition for AC and AE, respectively
	AC AE YS YF $\sigma_{c}^{bdt}, \sigma_{ex}^{bdt}$ $\psi_{c}^{bdt}, \psi_{ex}^{bc}$ $\bar{\tau}$	In the principal stress space axisymmetric compression axisymmetric extension yield surface yield function mean stress at brittle-ductile transition mean stress at brittle-ductile transition for AC and AE, respectively ψ at brittle-ductile transition for AC and AE, respectively von Mises stress
$\bar{\tau}_{c}(\sigma)$, $\bar{\tau}_{ex}(\sigma)$ initial yield functions for AC and AE, respectively σ_{0} mean stress at the intersection of $\bar{\tau}_{c}(\sigma)$, $\bar{\tau}_{ex}(\sigma)$	AC AE YS YF $\sigma_{c}^{bdt}, \sigma_{ex}^{bdt}$ $\psi_{c}^{bdt}, \psi_{ex}^{bc}$ $\bar{\tau}$ F	In the principal stress space axisymmetric compression axisymmetric extension yield surface yield function mean stress at brittle-ductile transition mean stress at brittle-ductile transition for AC and AE, respectively ψ at brittle-ductile transition for AC and AE, respectively von Mises stress yield function
$\sigma_0 \qquad \text{respectively} \\ \sigma_0 \qquad \text{mean stress at the intersection of } \bar{\tau}_c(\sigma), \ \bar{\tau}_{\text{ex}}(\sigma) \\$	AC AE YS YF $\sigma_{c}^{bdt}, \sigma_{ex}^{bdt}$ $\psi_{c}^{bdt}, \psi_{ex}^{bd}$ $\bar{\tau}$ F Φ	In the principal stress space axisymmetric compression axisymmetric extension yield surface yield function mean stress at brittle-ductile transition mean stress at brittle-ductile transition for AC and AE, respectively ψ at brittle-ductile transition for AC and AE, respectively von Mises stress yield function plastic potential function
σ_0 mean stress at the intersection of $\bar{\tau}_c(\sigma)$, $\bar{\tau}_{ex}(\sigma)$	AC AE YS YF $\sigma_{c}^{bdt}, \sigma_{ex}^{bdt}$ $\psi_{c}^{bdt}, \psi_{ex}^{bd}$ $\bar{\tau}$ F Φ $\bar{\tau}_{c}(\sigma), \bar{\tau}_{e}$	In the principal stress space axisymmetric compression axisymmetric extension yield surface yield function mean stress at brittle-ductile transition mean stress at brittle-ductile transition for AC and AE, respectively ψ at brittle-ductile transition for AC and AE, respectively von Mises stress yield function plastic potential function $_{x}(\sigma)$ initial yield functions for AC and AE,
	AC AE YS YF $\sigma_{c}^{bdt}, \sigma_{ex}^{bdt}$ $\psi_{c}^{bdt}, \psi_{ex}^{bd}$ $\bar{\tau}$ F Φ $\bar{\tau}_{c}(\sigma), \bar{\tau}_{e}$	In the principal stress space axisymmetric compression axisymmetric extension yield surface yield function mean stress at brittle-ductile transition mean stress at brittle-ductile transition for AC and AE, respectively ψ at brittle-ductile transition for AC and AE, respectively von Mises stress yield function plastic potential function $x(\sigma)$ initial yield functions for AC and AE, respectively

$\bar{\tau}_c^{pk}$, $\bar{\tau}_{ex}^{pk}$	von Mises stress at stress peaks for AC and AE
	loading, respectively
P_c	confining pressure in conventional tests
P^*	confining pressure in hydrostatic tests at the
	onset of grain crushing
q_r	coefficients in Eq. (1); $(r = 1, 25)$
a_m	coefficients in the function $\sigma_1(\sigma_2, \sigma_3)$ given in
	the caption of Fig. 2; $(m = 1, 26)$
А, В, С	functions defined in Eq. (3).
<i>w</i> , <i>w</i> ₁ , <i>w</i>	₂ coefficients (exponents) in the yield function,
	Eq. (2).
α	internal friction coefficient
β	dilatancy factor
f_{ij}	equal to $\partial F/\partial \sigma_{ij}$
f_{σ}	equal to $\partial F/\partial \sigma$, defines the internal friction
	coefficient α
$f_{ar{ au}}$	equal to $\partial F/\partial \overline{\tau}$
$f_{ heta}$	equal to $\partial F/\partial \theta$
Ω_{ij}	equal to $\partial \theta / \sigma_{ij}$
g_{ij}	equal to $\partial \Phi / \partial \sigma_{ij}$
g_{σ}	equal to $\partial \Phi / \partial \sigma$, defines the dilatancy factor β
$\varepsilon_{ij}, \varepsilon_{ij}^{el}, \varepsilon_{ij}^{f}$	total, elastic, and inelastic strain tensors,
n	respectively
e_{ij}^{P}	inelastic strain deviator tensor
$\bar{\gamma}^{p}$	accumulated inelastic equivalent shear strain
α_0	parameter linking α and β in Eq. (10).
dλ	non-negative scalar function in the flow rule,
	Eq. (8).
H	hardening modulus
h = H/G	normalized hardening modulus
n _{cr}	critical hardening modulus <i>n</i> when deforma-
1.*	tion bands are parallel to σ_1 , Eq. (38).
n_{cr}^{+}	critical hardening modulus when deformation
ı, dn	Dands are parallel to σ_2 , Eq. (39).
n_{cr}^{up}	Critical hardening modulus for the Drucker-
4 h	Plagel model, Eq. (41).
Δn_{cr}	equal to $h_{cr} - h_{cr}^{cr}$
Δn_{cf}	equal to $n_{cr} - n_{cr}$
	I^{p} total elastic and inelastic stiffness tonsors
Lijkl, Lijkl,	L_{ijkl} total, clastic, and metastic summess tensors (i i k l=1 2 3)
L. I* ~	$(i, j, \kappa, i = 1, 2, 3)$
I_{kl}, I_{ij}, ω	, n ucinicu ili Eqs. (23)–(20).

stands for the peak stress values corresponding to the onset of the material rupture, and the subscripts "*c*" and "*ex*" are compression and extension, respectively. Using an exceptionally large data set for the low porosity Solnhofen limestone from axisymmetric compression (AC) and axisymmetric extension (AE) conventional tests conducted under different confining pressure P_c , Heard (1960) was the first to show that this is true only up to a certain value of σ . Above this value the relation between $\bar{\tau}_c^{pk}$ and $\bar{\tau}_{ex}^{pk}$ becoming greater than $\bar{\tau}_c^{pk}$. This author also demonstrated for the first time that the transition from brittle to ductile behavior under extension occurs at σ value, σ_{ex}^{bdt} , almost twice (~1.7) that under compression, σ_c^{bdt} (the superscript "*bdt*" stands for brittle-ductile transition). In other words, rock behavior under extension is much more brittle than under compression. Therefore the extension loading is more prone to fracture development even at high pressure. These fundamental discoveries did not receive much attention from the geomechanics community and until recently were not confirmed because the overwhelming majority of rock tests were limited to a single AC loading type. Heard's results were only confirmed 50 years later by Nguyen et al. (2011). These authors conducted a wide series of both AC and AE tests under various P_c on synthetic Granular Rock Analog Material (GRAM1) consisting of "welded" TiO₂ particles. The nature of GRAM1 is obviously very different from that of Solnhofen limestone and it has more than two orders of magnitude lower strength, but the mechanical behavior of these two materials is very similar including the $\sigma_{ex}^{bdt}/\sigma_c^{bdt}$ ratio,

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