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Dependence of rock properties on the Lode angle: Experimental data, constitutive model, and bifurcation analysis

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ABSTRACT

The overwhelming majority of experimental tests on rocks have only been conducted for a single value of the Lode angle θ corresponding to the axisymmetric compression (AC). There are now sufficiently extensive data sets from both AC and axisymmetric extension (AE) tests (corresponding to two extreme θ values) for two materials (synthetic rock analog GRAM1 and Solnhofen Limestone). These data cover a wide range of the confining pressure (from brittle faulting to ductile flow). Very recently the data from true 3-D tests (for different θ) also covering both brittle and ductile fields were published for Castlegate and Bentheim Sandstone as well. The results from all these tests summarized and processed in this paper constitute a solid basis which allows general conclusions to be drawn about the dependence of rock behavior on θ. In all cases, the yield/failure envelopes were shown to be θ-dependent so that the material strength at low mean stress σ is smaller under AE than under AC, while at high σ, it is the opposite. The brittle-ductile transition under AE occurs at σ ~1.5 times greater than under AC, meaning that under AE the material is more prone to fracture development. The angle between the most compressive stress and the forming deformation localization bands is systematically higher for AE than for AC for the same σ. Based on these data we formulate a new three-invariant constitutive model with convex and concave yield functions (YFs) which is used for the bifurcation analysis. The results of this analysis agree with the experimental data (for both YFs) and reveal that the θ-dependence of rock properties encourages the strain localization. The major factors defining this dependence are the θ-dependence of the YFs but also of the dilatancy factor which is greater for AE than for AC. The theoretical results show that the failure (deformation band) plane can deviate from the intermediate stress direction and can become parallel to the maximum compressive stress at high σ for the concave YF.

1. Introduction

It has long been known that the strength of geomaterials under compression $\bar{\tau}_c^{pk}$ can be considerably higher than that under extension $\bar{\tau}_{ex}^{pk}$ for the same mean stress σ (Mogi, 1967, 1971; Chang and Haimson, 2000; Haimson and Chang, 2000; Haimson and Rudnicki, 2010; Haimson, 2011; Lee and Haimson, 2011), where $\bar{\tau}$ is the von Mises stress, the superscript "*pk*"

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Nomenclature

G	shear modulus
K	bulk modulus
E	Young's modulus
ν	Poisson's ratio
θ	Lode angle
σ_{ij}	stress tensor ($i, j = 1,\ 2,\ 3$)
σ_i	principal stresses
s_{ij}	stress deviator tensor
δ_{ij}	Kronecker delta
σ_1	maximum compression stress
σ_2	intermediate principal stress
σ	mean stress
σ_{cr}	mean stress at the crest of initial yield envelopes
J_3	third invariant of stress deviator
ψ	angle between σ_2-parallel deformation localization bands (planes) and σ_1 direction
ψ^*	angle between σ_1-parallel deformation bands and σ_2 direction
$\Delta\psi$	$\psi(\theta = 0°) \ - \ \psi(\theta = 60°)$
$\Delta\psi^B$, $\Delta\psi^C$	$\Delta\psi$ for Bentheim and Castlegate sandstones
$\Delta\psi^G$, $\Delta\psi^S$	$\Delta\psi$ for GRAM1 material, and Solnhofen limestone
ξ	equal to $\cos\ 2\psi$, Eq. (32).
n_i	unit normal to deformation localization bands in the principal stress space
AC	axisymmetric compression
AE	axisymmetric extension
YS	yield surface
YF	yield function
σ^{bdt}	mean stress at brittle-ductile transition
σ_c^{bdt}, σ_{ex}^{bdt}	mean stress at brittle-ductile transition for AC and AE, respectively
ψ_c^{bdt}, ψ_{ex}^{bdt}	ψ at brittle-ductile transition for AC and AE, respectively
$\bar{\tau}$	von Mises stress
F	yield function
Φ	plastic potential function
$\bar{\tau}_c(\sigma)$, $\bar{\tau}_{ex}(\sigma)$	initial yield functions for AC and AE, respectively
σ_0	mean stress at the intersection of $\bar{\tau}_c(\sigma)$, $\bar{\tau}_{ex}(\sigma)$
$\bar{\tau}_c^{pk}$, $\bar{\tau}_{ex}^{pk}$	von Mises stress at stress peaks for AC and AE loading, respectively
P_c	confining pressure in conventional tests
P^*	confining pressure in hydrostatic tests at the onset of grain crushing
q_r	coefficients in Eq. (1); ($r = 1, 2...5$)
a_m	coefficients in the function $\sigma_1(\sigma_2, \sigma_3)$ given in the caption of Fig. 2; ($m = 1, 2...6$)
A, B, C	functions defined in Eq. (3).
w, w_1, w_2	coefficients (exponents) in the yield function, Eq. (2).
α	internal friction coefficient
β	dilatancy factor
f_{ij}	equal to $\partial F/\partial \sigma_{ij}$
f_σ	equal to $\partial F/\partial \sigma$, defines the internal friction coefficient α
$f_{\bar{\tau}}$	equal to $\partial F/\partial \bar{\tau}$
f_θ	equal to $\partial F/\partial \theta$
Ω_{ij}	equal to $\partial \theta/\sigma_{ij}$
g_{ij}	equal to $\partial \Phi/\partial \sigma_{ij}$
g_σ	equal to $\partial \Phi/\partial \sigma$, defines the dilatancy factor β
ε_{ij}, ε_{ij}^{el}, ε_{ij}^{p}	total, elastic, and inelastic strain tensors, respectively
e_{ij}^p	inelastic strain deviator tensor
$\bar{\gamma}^p$	accumulated inelastic equivalent shear strain
α_0	parameter linking α and β in Eq. (10).
$d\lambda$	non-negative scalar function in the flow rule, Eq. (8).
H	hardening modulus
$h = H/G$	normalized hardening modulus
h_{cr}	critical hardening modulus h when deformation bands are parallel to σ_1, Eq. (38).
h_{cr}^*	critical hardening modulus when deformation bands are parallel to σ_2, Eq. (39).
h_{cr}^{dp}	critical hardening modulus for the Drucker-Prager model, Eq. (41).
Δh_{cr}	equal to h_{cr}–h_{cr}^*
Δh_{cr}^{dp}	equal to h_{cr}–h_{cr}^{dp}
Q and R	defined in Eqs. (31), (35), (36).
L_{ijkl}, L_{ijkl}^{el}, L_{ijkl}^{p}	total, elastic, and inelastic stiffness tensors (i, j, k, l=1, 2, 3)
I_{kl}, I_{ij}^*, ω, Λ	defined in Eqs. (23)–(28).

stands for the peak stress values corresponding to the onset of the material rupture, and the subscripts "*c*" and "*ex*" are compression and extension, respectively. Using an exceptionally large data set for the low porosity Solnhofen limestone from axisymmetric compression (AC) and axisymmetric extension (AE) conventional tests conducted under different confining pressure P_c, Heard (1960) was the first to show that this is true only up to a certain value of σ. Above this value the relation between $\bar{\tau}_c^{pk}$ and $\bar{\tau}_{ex}^{pk}$ is inversed, $\bar{\tau}_{ex}^{pk}$ becoming greater than $\bar{\tau}_c^{pk}$. This author also demonstrated for the first time that the transition from brittle to ductile behavior under extension occurs at σ value, σ_{ex}^{bdt}, almost twice ($\sim$1.7) that under compression, σ_c^{bdt} (the superscript "*bdt*" stands for brittle-ductile transition). In other words, rock behavior under extension is much more brittle than under compression. Therefore the extension loading is more prone to fracture development even at high pressure. These fundamental discoveries did not receive much attention from the geomechanics community and until recently were not confirmed because the overwhelming majority of rock tests were limited to a single AC loading type. Heard's results were only confirmed 50 years later by Nguyen et al. (2011). These authors conducted a wide series of both AC and AE tests under various P_c on synthetic Granular Rock Analog Material (GRAM1) consisting of "welded" TiO_2 particles. The nature of GRAM1 is obviously very different from that of Solnhofen limestone and it has more than two orders of magnitude lower strength, but the mechanical behavior of these two materials is very similar including the $\sigma_{ex}^{bdt}/\sigma_c^{bdt}$ ratio,