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Distributional and regularized radiation fields of non-uniformly moving straight dislocations, and elastodynamic Tamm problem

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ABSTRACT

This work introduces original explicit solutions for the elastic fields radiated by non-uniformly moving, straight, screw or edge dislocations in an isotropic medium, in the form of time-integral representations in which acceleration-dependent contributions are explicitly separated out. These solutions are obtained by applying an isotropic regularization procedure to distributional expressions of the elastodynamic fields built on the Green tensor of the Navier equation. The obtained regularized field expressions are singularity-free, and depend on the dislocation density rather than on the plastic eigenstrain. They cover non-uniform motion at arbitrary speeds, including faster-than-wave ones. A numerical method of computation is discussed, that rests on discretizing motion along an arbitrary path in the plane transverse to the dislocation, into a succession of time intervals of constant velocity vector over which time-integrated contributions can be obtained in closed form. As a simple illustration, it is applied to the elastodynamic equivalent of the Tamm problem, where fields induced by a dislocation accelerated from rest beyond the longitudinal wave speed, and thereafter put to rest again, are computed. As expected, the proposed expressions produce Mach cones, the dynamic build-up and decay of which is illustrated by means of full-field calculations.

1. Introduction

Dislocations are line defects whose motion is responsible for plastic deformation in crystalline materials (Hirth and Lothe, 1982). To improve the current understanding of the plastic and elastic fronts (Clifton and Markenscoff, 1981) that go along with extreme shock loadings in metals (Meyers et al., 2009), Gurrutxaga-Lerma et al. (2013) have recently proposed dynamic simulations of large sets of dislocations mutually coupled by their retarded elastodynamic field. Gurrutxaga-Lerma et al. (2014) review the matter and its technical aspects in some detail. This new approach is hoped to provide complementary insights over multi-physics large-scale atomistic simulations of shocks in matter (Zhakhovsky et al., 2011). If we leave aside the subsidiary (but physically important) issue of dislocation nucleation, dislocation-dynamics simulations involve two separate but interrelated tasks. First, one needs to compute the field radiated by a dislocation that moves arbitrarily. Second, given the past history of each dislocation, the current dynamic stress field incident on it due to the other

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ones, and the externally applied stress field (e.g., a shock-induced wavefront), the further motion of the dislocation must be determined by a dynamic mobility law. While some progress has recently been achieved in the latter subproblem—which involves scarcely explored radiation-reaction effects and dynamic core-width variations (Pellegrini, 2014)—the focus of the present paper is on the former—a very classical one.

Indeed, substantial effort has been devoted over decades to obtaining analytical expressions of elastodynamic fields produced by non-uniformly moving singularities such as point loads (Stronge, 1970; Freund, 1972, 1973), cracks, and dislocations. Results ranged, e.g., from straightforward applications to linear-elastic and isotropic unbounded media, to systems with interfaces such as half-spaces (Lamb's problem) or layered media (Eatwell et al., 1982); coupled phenomena such as thermoelasticity (Brock et al., 1997) or anisotropic elastic media (Markenscoff and Ni, 1987; Wu, 2000), to mention but a few popular themes. Elastodynamic fields of dislocations have been investigated in a large number of works, among which Eshelby (1951), Kiusalaas and Mura (1964, 1965), Mura (1987), Nabarro (1967), Brock (1979, 1982, 1983), Markenscoff (1980), Markenscoff and Ni (2001a,b), Pellegrini (2010) and Lazar (2011b, 2012, 2013a,b). Early numerical implementations of time-dependent fields radiated by moving sources (Niazy, 1975; Madariaga, 1978) were limited to material displacements or velocities. As to stresses, Gurrutxaga-Lerma et al. (2014) based their simulations on the fields of Markenscoff and Clifton (1981) relative to a subsonic edge dislocation. Nowadays dynamic fields of individual dislocations or cracks are also investigated by atomistic simulations (Li and Shi, 2002; Tsuzuki et al., 2009; Spielmannová et al., 2009), or numerical solutions of the wave equation by means of finite-element (Zhang et al., 2015), finite-difference, or boundary-integral schemes (Day et al., 2005). Hereafter, the analytical approach is privileged so as to produce reference solutions.

Disregarding couplings with other fields such as temperature, one might be tempted to believe that the simplest two-dimensional problem of the non-uniform motion of rectilinear dislocation lines in an unbounded, linear elastic, isotropic medium, leaves very little room for improvements over past analytical works. This is not so, and our present concerns are as follows:

(i) *Subsonic as well as supersonic velocities.* In elastodynamics, from the 1970's onwards, the method of choice for analytical solutions has most often been the one of Cagniard improved by de Hoop (Aki and Richards, 2009), whereby Laplace transforms of the fields are inverted by inspection after a deformation of the integration path of the Laplace variable has been carried out by means of a suitable change of variable (see above-cited references). However, to the best of our knowledge, no such solutions can be employed indifferently for subsonic and supersonic motions, in the sense that the supersonic case need be considered separately in order to get explicit results as, e.g., in Stronge (1970), Freund (1972), Callias and Markenscoff (1980), Markenscoff and Ni (2001b) and Huang and Markenscoff (2011). Indeed, carrying out the necessary integrals usually requires determining the wavefront position relatively to the point of observation. To date, the supersonic edge dislocation coupled to both shear and longitudinal waves has not been considered, and existing supersonic analytical solutions for the screw dislocation have not proved usable in full-field calculations, except for the rather different solution obtained within the so-called gauge-field theory of dislocations (Lazar, 2009), which appeals to gradient elasticity. Thus, one objective of the present work is to provide 'automatic' theoretical expressions that do not require wavefront tracking, for both screw and edge dislocations, and can be employed whatever the dislocation velocity. To this aim, we shall employ a method different from the Cagniard–de Hoop one. This is not to disregard the latter but following a different route was found more convenient in view of the remaining points listed here.

(ii) *Distributions and smooth regularized fields.* For a Volterra dislocation in supersonic steady motion, fields are typically concentrated on Dirac measures along infinitely thin lines, to form Mach cones (Stronge, 1970; Callias and Markenscoff, 1980; Weertman and Weertman, 1980). Thus, the solution is essentially of distributional nature, and its proper characterization involves, beside Dirac measures, the use of principal-value and finite-parts prescriptions (Pellegrini and Lazar, 2015). Of course, in-depth analytical characterizations of wavefronts singularities can still be extracted out of Laplace-transform integral representations (Freund, 1972, 1973; Callias and Markenscoff, 1980). However their distributional character implies that the solutions cannot deliver meaningful numbers unless they are regularized by convolution with some source shape function representing a dislocation of finite width. Only by this means can field values in Mach cones be computed. Consequently, another objective is to provide field expressions for an extended dislocation of finite core width (instead of a Volterra one), thus taming all the field singularities that would otherwise be present at wavefronts and at the dislocation location, where Volterra fields blow up. In the work by Gurrutxaga-Lerma et al. (2013), a simple cut-off procedure was employed to get rid of infinities. Evidently, a similar device cannot be used with Dirac measures, which calls for a smoother and more versatile regularization. Various dislocation-regularizing devices have been proposed in the past, some consisting in expanding the Volterra dislocation into a flat Somigliana dislocation (Eshelby, 1949, 1951; Markenscoff and Ni, 2001a,b; Pellegrini, 2011). Such regularizations remove infinities, but leave out field discontinuities on the slip path (Eshelby, 1949). A smoother approach consisting in introducing some non-locality in the field equations has so far only be applied to the time-dependent motion of a screw dislocation. The one to be employed hereafter, introduced in Pellegrini and Lazar (2015), achieves an isotropic expansion the Volterra dislocation and smoothly regularizes all field singularities for screw and edge dislocations. In this respect, it resembles that introduced in statics by Cai et al. (2006). However, we believe it better suited to dynamics.

(iii) *Field-theoretic framework.* The traditional method of solution (Markenscoff, 1980) rests on imposing suitable boundary conditions on the dislocation path. It makes little contact with field-theoretic notions of dislocation theory such as plastic strain, or dislocation density and current used in purely numerical methods of solution (Djaka et al., 2015). Instead, we wish our analytical results to be rooted on a field-theoretic background. One advantage is that the approach will provide