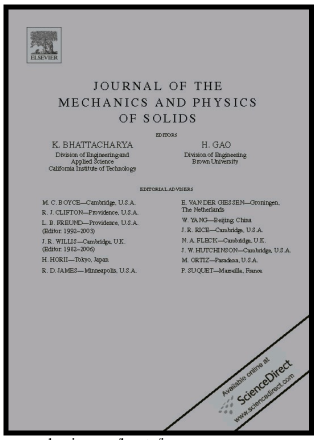
Author's Accepted Manuscript



Fracture propagation in brittle materials as a standard dissipative process: General theorems and crack tracking algorithms

A. Salvadori, F. Fantoni

PII: S0022-5096(15)30253-2
DOI: http://dx.doi.org/10.1016/j.jmps.2016.04.034
Reference: MPS2887

To appear in: *Journal of the Mechanics and Physics of Solids*

Received date: 12 November 2015
Revised date: 22 April 2016
Accepted date: 22 April 2016

Cite this article as: A. Salvadori and F. Fantoni, Fracture propagation in brittle materials as a standard dissipative process: General theorems and crack tracking algorithms, *Journal of the Mechanics and Physics of Solids* http://dx.doi.org/10.1016/j.jmps.2016.04.034

Fracture propagation in brittle materials as a standard dissipative process: general theorems and crack tracking algorithms.

A. Salvadori[1,2,3], F. Fantoni[1]
[1] DICATAM, Università di Brescia, via Branze 43, 25123 Brescia, Italy
[2] Department of Aerospace and Mechanical Engineering,
[3] Center for Shock Wave-processing of Advanced Reactive Materials,
University of Notre Dame, Notre Dame, Indiana.

Abstract

The present work frames the problem of three-dimensional quasi-static crack propagation in brittle materials into the theory of standard dissipative processes. Variational formulations are stated. They characterize the three dimensional crack front "quasi-static velocity" as minimizer of constrained quadratic functionals. An implicit in time crack tracking algorithm that computationally handles the constraint via the penalty method algorithm is introduced and proof of concept provided.

Keywords : Brittle materials, Crack growth, Variational formulations, Penalty method

1 Introduction

Cracks can develop and propagate catastrophically in brittle materials, with little or no warning. The copious literature devoted to model the initiation and growth of fracture confirm the importance of understanding crack pattern evolution and of predicting quantities of interest in order to increase the safety and improve the mechanical performance of materials and components.

On the other hand, controlled quasi static cracks growth can be beneficial, as for the generation of new surface area in reactive materials, or for hydraulic fracturing in shale gas extraction.

Either positive or negative the interpretation of crack growth, the major complexity stands in governing fracture propagation in brittle or embrittled materials before unstable propagation takes place. In other words, it concerns the ability of quantitatively predicting the condition of stable/unstable transition. A precise formulation of the problem can be stated as follows: given the state of stress and the history of crack propagation at a given time, express the crack propagation rate of the crack front due to a given variation of external actions. This formulation will be termed *the global (incremental) quasi-static fracture propagation problem*.

Variationally-based numerical schemes [1, 2] and phase field methods [3, 4] have undergone great developments, with applications in various multi-physics context [5], and are nowadays standard techniques to deal with the global quasi-static fracture propagation problem. These approaches typically employ a single order parameter to avoid explicit crack front tracking. The present note differs in this regard. It frames the problem of crack propagation in brittle materials into the theory of standard dissipative processes [6] and ultimately leads to formulate algorithms for crack propagation that do not seek for global minima and do not require any regularization parameter.

As in Griffith's theory [7] a local condition along the crack front rules the onset of crack propagation. It describes when the process region attains a critical state, which is independent of body, loading and geometry in most cases of engineering interest. The latter property is termed autonomy [8].

Several scientific publications put Linear Elastic Fracture Mechanics (LEFM) in analogy with the thermodynamics of standard dissipative processes [9, 10, 11]. The first part of the present note provides a systematic