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A model for extreme plasticity

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ABSTRACT

We present a mathematical model for elastoplasticity in the regime where the applied stress greatly exceeds the yield stress. This scenario is typically found in violent impact testing, where millimetre thick metal samples are subjected to pressures on the order of $10–10^2$ GPa, while the yield stress can be as low as 10^{-2} GPa. In such regimes the metal can be treated as a barotropic compressible fluid in which the strength, measured by the ratio of the yield stress to the applied stress, is negligible to lowest order. Our approach is to exploit the smallness of this ratio by treating the effects of strength as a small perturbation to a leading order barotropic model. We find that for uniaxial deformations, these additional effects give rise to features in the response of the material which differ significantly from the predictions of barotropic flow.

1. Introduction

Most simulations of the mechanical response of a metal undergoing violent elastic–plastic deformation rely on knowledge of the equation of state (EoS) for the material under study. Traditionally, shock waves generated using a gas-gun have been used to determine this information theoretically and experimentally (Davison and Graham, 1979; Molinari and Ravichandran, 2004; Meyers, 1994; Davison, 2008; Clifton, 1985; Germain and Lee, 1973). More recently, the so-called isentropic compression experiments (ICEs) have become a standard method by which we extract EoS information in the absence of shock waves (Davis, 2006; Rothman et al., 2014, 2005). There are several advantages in using ICEs over shock wave experiments, one of which is that the entire isentrope can be obtained in a single experiment. In comparison, one shock wave experiment gives only one point on the Hugoniot, corresponding to a single value of the entropy. Multiple experiments are then required to generate the entire EoS. Another advantage is that the temperature in shock wave experiments can become sufficiently high to melt the material, while in ICEs the material remains in the solid phase. This allows EoS data for the solid phase to be obtained at much higher pressures.

A schematic of a typical experiment is shown in Fig. 1. Using a magnetic pressure drive, a ramped compression wave is made to propagate through the target sample. The target is designed to be thin compared to its lateral extent, so that waves generated at the outer edge of the sample do not have time to reach the centre during the time-scale of the experiment. Thus the material at the centre of the sample undergoes purely uniaxial deformation, with displacement varying only in the direction of impact. After a short time, the velocity at the rear face of the target material is recorded using velocity interferometry. The results from a typical experiment are shown in Fig. 2. When an attempt is made to reconstruct this velocity profile numerically by tuning certain parameters in the constitutive assumptions of the model, the problem becomes an

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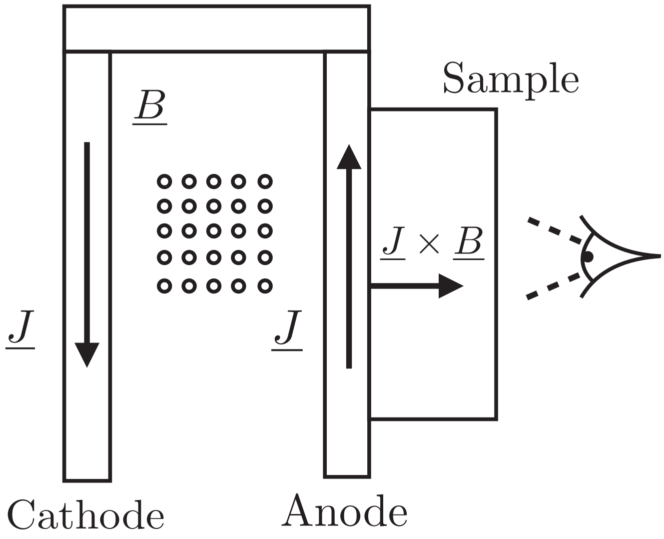

Fig. 1. Schematic of a ramped isentropic compression experiment. A current passes between the cathode and anode, perpendicular to a magnetic field directed into the page. The resulting Lorentz force provides a pressure drive against the front face of the target sample.

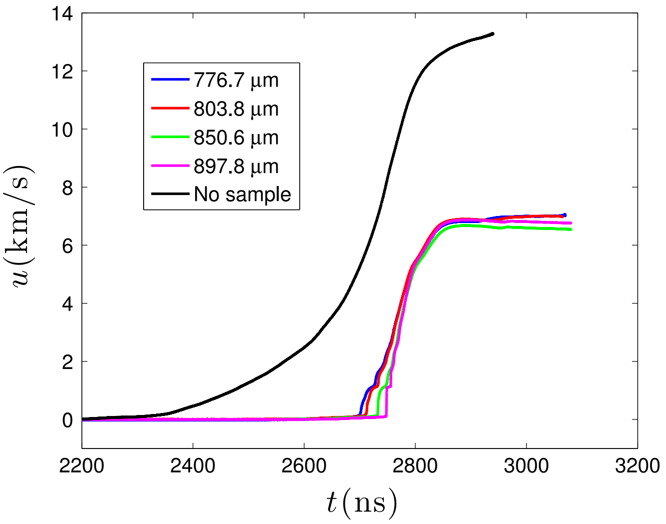

Fig. 2. Boundary velocimetry data obtained from 3 MBar compression of lead (Rothman et al., 2014). The coloured curves show the free-surface velocity u measured for different thicknesses of the sample. A reference curve with no lead sample is plotted in black. (For interpretation of the references to colour in this figure caption, the reader is referred to the web version of this paper.)

inverse problem for the equation of state.

Previous attempts to infer EoS information from such experiments have typically neglected the effects of mechanical strength, instead treating the material as a barotropic compressible fluid (Hinch, 2010; Ockendon et al., 2010). This approximation is based on the fact that, under extreme conditions, the ability of a solid object to resist shear is limited by the yield stress, which is typically much smaller than the applied stress. However, violent impact experiments reported by numerous authors have confirmed the existence of both elastic and plastic waves (Meyers, 1994; Clifton, 1985; Pack et al., 1948; Von Karman and Duwez, 1950; Whitley et al., 2011). The former propagate through the material as the stress increases toward the yield stress, and the latter as the material is compressed further beyond the yield surface. Therefore, a proper account of violent elastic–plastic deformation requires one to account for both the compressibility of the material and the small but measurable effects of elasticity.

There exist numerous macroscopic models in the literature pertaining to metal plasticity, which reflects the reality that no one theory of plasticity is universally accepted, in contrast to the theories of elasticity or fluid dynamics (Steinberg et al., 1980; Steinberg and Lund, 1989; Green and Naghdi, 1965; Willis, 1969; Howell et al., 2016, 2014; Plohr and Sharp, 1992). The key physical phenomenon underpinning plastic deformation is the nucleation and motion of dislocations (Orowan et al., 1954; Hirth and Lothe, 1982; Clifton and Markenscoff, 1981; Johnson and Barker, 1969). However, even on the length-scale of