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Dynamic bulk and shear moduli due to grain-scale local fluid flow in fluid-saturated cracked poroelastic rocks: Theoretical model

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ABSTRACT

Grain-scale local fluid flow is an important loss mechanism for attenuating waves in cracked fluid-saturated poroelastic rocks. In this study, a dynamic elastic modulus model is developed to quantify local flow effect on wave attenuation and velocity dispersion in porous isotropic rocks. The Eshelby transform technique, inclusion-based effective medium model (the Mori–Tanaka scheme), fluid dynamics and mass conservation principle are combined to analyze pore-fluid pressure relaxation and its influences on overall elastic properties. The derivation gives fully analytic, frequency-dependent effective bulk and shear moduli of a fluid-saturated porous rock. It is shown that the derived bulk and shear moduli rigorously satisfy the Biot-Gassmann relationship of poroelasticity in the low-frequency limit, while they are consistent with isolated-pore effective medium theory in the high-frequency limit. In particular, a simplified model is proposed to quantify the squirt-flow dispersion for frequencies lower than stiff-pore relaxation frequency. The main advantage of the proposed model over previous models is its ability to predict the dispersion due to squirt flow between pores and cracks with distributed aspect ratio instead of flow in a simply conceptual double-porosity structure. Independent input parameters include pore aspect ratio distribution, fluid bulk modulus and viscosity, and bulk and shear moduli of the solid grain. Physical assumptions made in this model include (1) pores are inter-connected and (2) crack thickness is smaller than the viscous skin depth. This study is restricted to linear elastic, well-consolidated granular rocks.

1. Introduction

The presence of microscale cracks can have a large effect on the effective elastic properties of a solid even if the fractional volume occupied by the cracks is extremely small. The state of fluid saturation of the cracks has a corresponding large effect. When a cracked rock is compressed by passing waves, the fluid pressure response in the compliant/soft cracks will be greater than that in the stiffer pores. The induced pressure gradient creates grain-scale local flow from crack to adjacent pores. The resulting fluid flow is called "squirt flow (SF)" (e.g., Mavko and Nur, 1975) which also occurs in shear waves. In

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Nomenclature

ϕ	total porosity
ϕ_c	total crack porosity
ϕ_p	total stiff-pore porosity
$\phi^i, \phi_c^i, \phi_p^i$	porosity of the ith pore, crack, stiff pore
γ^i	aspect ratio of the ith crack
γ_p	aspect ratio of the stiff pores
v	volume of a rock sample
v_ϕ	volume of the total pore space
v_c^i, v_p^i	volume of the ith crack, stiff pore
v_p	volume of the total stiff pores
$\mathbf{L}_0, K_0, G_0$	elastic modulus tensor, bulk and shear moduli of solid grain
ν_0	Poisson's ratio of the solid grain
$\mathbf{L}_f, K_f$	elastic modulus tensor, bulk modulus of pore fluid
ρ_f, η	density, viscosity of pore fluid
$\mathbf{L}_d, K_d, G_d$	elastic modulus tensor, bulk and shear moduli of dry sample
$\mathbf{L}, K, G$	effective elastic modulus tensor, bulk and shear moduli
f, ω	frequency, circular frequency
$\mathbf{S}^i, \mathbf{S}_c^i, \mathbf{S}_p^i$	Eshelby tensor of the ith pore, crack, stiff pore
$\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_c^i, \boldsymbol{\sigma}_p^i$	averaged fluid stress tensor in the ith pore, crack, stiff pore
$\mathbf{e}_c^i, \mathbf{e}_p^i$	strain tensor of the ith crack, stiff pore
$\mathbf{e}_{cf}^i, \mathbf{e}_{pf}^i$	fluid strain tensor in the ith crack, stiff pore
q_c^i, q_p^i	volumetric flow out of the ith crack, stiff pore
q_p	averaged volumetric flow over stiff pores
$\mathbf{e}_\phi$	averaged pore strain over all pore space
$\mathbf{e}_f$	averaged fluid strain over all pore space
$\mathbf{e}_\infty$	externally applied strain
$\boldsymbol{\sigma}_f$	averaged pore fluid stress tensor over all pore space
p_c^i, p_p^i	averaged fluid pressure in the ith crack, stiff pore
$\tilde{p}_c^i$	deformation-induced fluid pressure in the ith crack
p_p	averaged fluid pressure over stiff pores
p_f	averaged fluid pressure over all pores
r, θ, z	cylindrical coordinates
v_r, v_θ, v_z	fluid velocity components in a crack
$\mathbf{u}_f$	fluid particle displacement vector in a crack
$k_r(=\tilde{k}), k_z$	wavenumber components in a crack
f_c^i	relaxation function of the ith crack
N_p	number of stiff pores
S_c	upper and lower surfaces of a crack
$\mathbf{n}$	normal vector directed outward from crack fluid
h, R	thickness, radius of a crack
L	effective distance between stiff pores
S	effective flow cross section area of stiff pores
δ	dimensionless constant for stiff pores
$\mathbf{Q}_c^i, \mathbf{Q}_p^i$	transform tensor relating $\mathbf{e}_\infty$ to q_c^i, q_p^i
$\mathbf{D}_c^i, \mathbf{D}_p^i, \mathbf{C}$	transform tensor relating $\mathbf{e}_\infty$ to $\boldsymbol{\sigma}_c^i, \boldsymbol{\sigma}_p^i, \boldsymbol{\sigma}_f$
$\tilde{\mathbf{T}}_c^i, \tilde{\mathbf{T}}_p^i, \tilde{\mathbf{T}}_\phi$	transform tensor relating $\mathbf{e}_\infty$ to $\mathbf{e}_c^i, \mathbf{e}_p^i, \mathbf{e}_\phi$
$\mathbf{T}_c^i, \mathbf{T}_p^i$	transform tensor relating $\mathbf{e}_\infty$ to $\mathbf{e}_c^i, \mathbf{e}_p^i$ for isolated pores

turn, the SF attenuates waves and changes effective elastic properties of rocks (e.g., David et al., 2013). Previous studies show that the SF appears to be an important physical mechanism in attenuating waves at sonic and ultrasonic frequencies (Mavko and Nur, 1975, 1979; Jones, 1986). Compared with Biot's global flow mechanism (Biot, 1956a, b), in which viscous-fluid motion is induced by the wavelength-scale fluid pressure gradient, the SF is a consequence of pore microstructural geometry and can produce much greater phase velocity dispersion and wave attenuation. The SF can give rise to significant hydrocarbon signature in the acoustic measurement (Tang et al., 2012) and affect the borehole signals (Markova et al., 2013). Over the years, many efforts have been made to develop unified models of global flow and squirt flow for dynamics of macroscopically homogeneous porous rocks having grain-scale cracks. Those include works by Dvorkin and Nur (1993), Jakobsen and Chapman (2009), Gurevich et al. (2010), Tang (2011) and Tang et al. (2012). We restrict this discussion to situations in which the Biot global flow mechanism is neglected.

In the low-frequency limit, the pore fluid pressure is in equilibrium. The elastic properties of rocks are described by Gassmann's equations (Gassmann, 1951), in which undrained (or relaxed) bulk modulus are calculated from drained (or dry) bulk modulus, solid grain bulk modulus, fluid bulk modulus and porosity, while undrained shear modulus is identical to drained shear modulus. Experimental data presented by Thomsen (1985) suggests that the Gassmann's equations are indeed satisfied for a wide range of different rock types. Pride and Berryman (1998) related variables controlled and measured in elastostatic laboratory experiments to the appropriate variables of poroelastic theory. With increasing frequency, induced fluid pressure in cracks stiffens rocks, resulting in frequency-dependent velocities.

In the high-frequency limit (in which wavelength is required to be still larger than porous representative volume element), the fluid pressure does not have enough time to equilibrate between pores. Then no fluid mass communication occurs so that all pores behave like isolated inclusions. It is more appropriate to estimate the unrelaxed effective elastic moduli using effective medium theory. Recently, David and Zimmerman (2012) developed a procedure to extract the pore aspect ratio distribution from the dry velocities. Their results showed that for ultrasonic velocity measurements, the predictions of saturated velocities using Mori–Tanaka (MT) scheme (Mori and Tanaka, 1973; Benveniste, 1987) and differential effective medium (LeRavalec and Guéguen, 1996) matched well the experimental data for a good number of sandstone data sets.

On the other hand, for a certain porous sample, its pore structure cannot change with frequency. The SF dispersion is solely due to pore-microstructure-induced fluid pressure relaxation. Because of this, the effective medium schemes are also widely used to estimate the static undrained elastic moduli (e.g., Berryman, 1981; Xu and White, 1995; LeRavalec and