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Strength and fracture toughness of heterogeneous blocks with joint lognormal modulus and failure strain

Leon S. Dimas [a,b], Daniele Veneziano [b], Markus J. Buehler [a,b,*]

[a] Laboratory for Atomistic and Molecular Mechanics (LAMM), Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA, USA
[b] Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA, USA

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ABSTRACT

We obtain analytical approximations to the probability distribution of the fracture strengths of notched one-dimensional rods and two-dimensional plates in which the stiffness (Young's modulus) and strength (failure strain) of the material vary as jointly lognormal random fields. The fracture strength of the specimen is measured by the elongation, load, and toughness at two critical stages: when fracture initiates at the notch tip and, in the 2D case, when fracture propagates through the entire specimen. This is an extension of a previous study on the elastic and fracture properties of systems with random Young's modulus and deterministic material strength (Dimas et al., 2015a). For 1D rods our approach is analytical and builds upon the ANOVA decomposition technique of (Dimas et al., 2015b). In 2D we use a semi-analytical model to derive the fracture initiation strengths and regressions fitted to simulation data for the effect of crack arrest during fracture propagation. Results are validated through Monte Carlo simulation. Randomness of the material strength affects in various ways the mean and median values of the initial strengths, their log-variances, and log-correlations. Under low spatial correlation, material strength variability can significantly increase the effect of crack arrest, causing ultimate failure to be a more predictable and less brittle failure mode than fracture initiation. These insights could be used to guide design of more fracture resistant composites, and add to the design features that enhance material performance.

1. Introduction

The initiation and propagation of fracture in notched linear elastic media is controlled by the stresses, strains and material strength around the evolving fracture tip. In turn, the stresses and strains depend on the boundary conditions, the specimen geometry (including the prior fracture path), and the stiffness tensor field over the entire body.

Natural and synthetic materials often exhibit significant stochastic-looking fluctuations in elastic modulus and strength (Erickson and Wiltschko, 1991; Gupta et al., 2006; Mayo and Nix, 1988; Tai et al., 2007; Weibull, 1939; Younis et al., 2012). Several studies have found that fluctuations in the modulus can enhance the fracture performance of materials such as bone and nacre (Dimas et al., 2014; Fantner et al., 2005; Hang and Barber, 2011; Hang et al., 2014; Hu et al., 2015; Jaasma et al.,

* Corresponding author at: Laboratory for Atomistic and Molecular Mechanics (LAMM), Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA 02139, USA
E-mail address: mbuehler@MIT.EDU (M.J. Buehler).

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2002; Tai et al., 2007; Younis et al., 2012). However, depending on the pattern of the fluctuations, the opposite may also be true and when the fluctuations are stochastic, various weakening and strengthening outcomes are usually possible. Understanding the influence of random material heterogeneities on the bulk mechanical properties using analytical/numerical predictive models is a critical first step for the reliability assessment and design of such materials.

Previous research has focused on cases with random spatial variation of the elastic modulus or the local strength, but not both. Studies that include fluctuations of the elastic modulus use mostly numerical tools, typically variants of finite element analysis (Rahman and Rao, 2002; Rao and Rahman, 2002; Reddy and Rao, 2008) in combination with first and second order uncertainty propagation (Grigoriu et al., 1990; Madsen et al., 2006) or Monte Carlo simulation (Curtin, 1997; Sahimi and Arbabi, 1993). While these numerical methods are general, they are computationally inefficient and unsuited for parametric analysis, optimization or design.

Materials with random local variation of strength have also been analyzed. Following Weibull's pioneering work (Weibull, 1939), several studies view the failure of initially intact specimens as a weakest-link problem, which is often amenable to analytical treatment (Duxbury and Leath, 1994; Herrmann et al., 1989; Leath and Duxbury, 1994). Important analytical work has also been done on the fracture of random structures with significant crack growth prior to failure (so called quasi-brittle structures; e.g. Bazant and Xi, 1991a, b; Bazant and Chen, 1997; Karihaloo, 2007). For example, by analyzing the stress field immediately prior to fracture, analytical expressions for the failure probability can be found using non-local theory (Bazant and Xi, 1991a, b). Other studies consider specimens in which fracture initiates at the tip of a pre-existing notch and focus on the fracture path using numerical simulation (Beyerlein and Phoenix, 1997; Chudnovsky and Kunin, 1987; Roux et al., 2003). We are not aware of other analytical investigations in which randomness of the bulk fracture

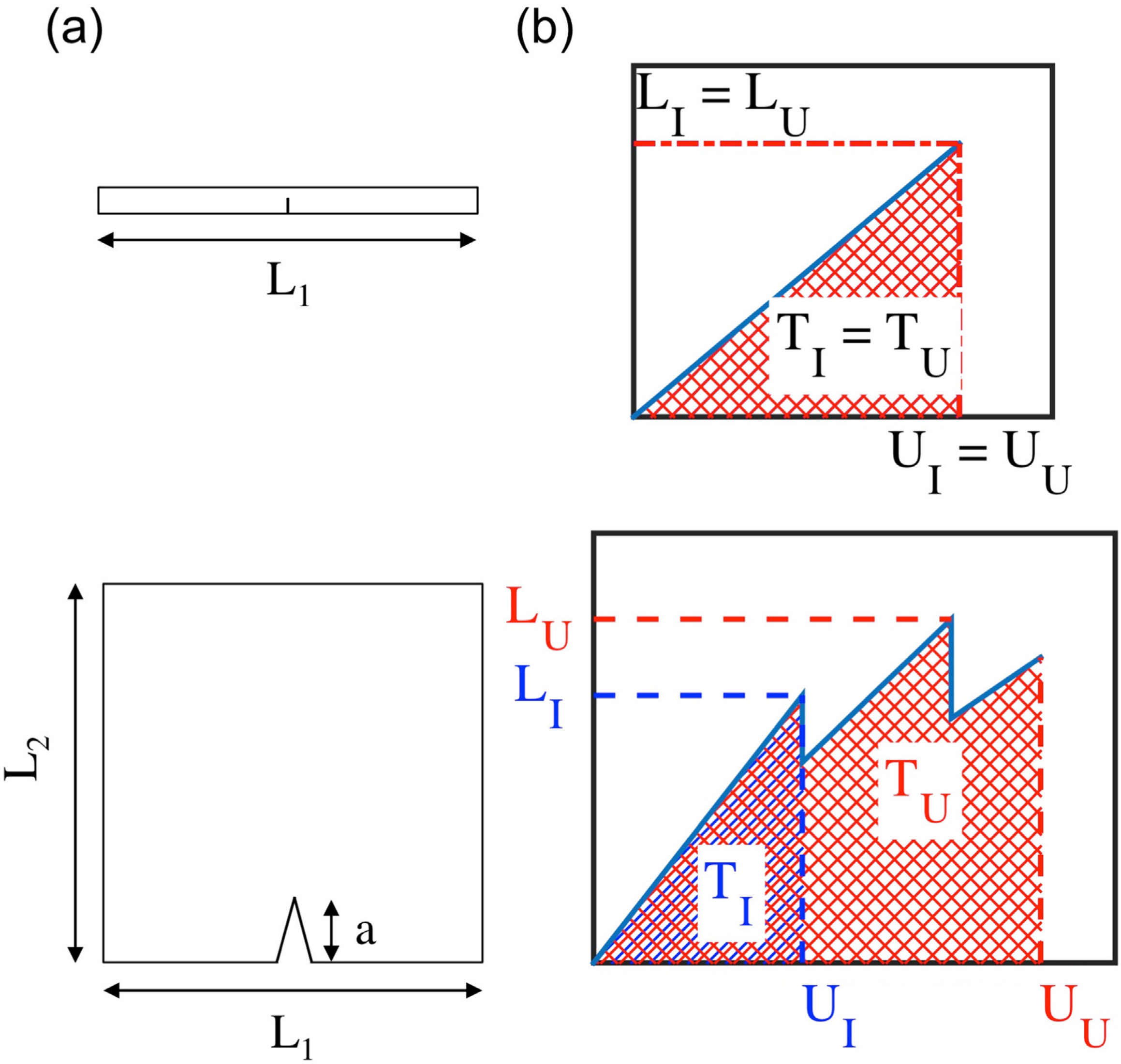

Fig. 1. (a) Specimen geometry and, (b) fracture strength measures for 1D rods and 2D plates.