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The destabilizing effect of external damping: Singular flutter boundary for the Pflüger column with vanishing external dissipation

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ABSTRACT

Elastic structures loaded by non-conservative positional forces are prone to instabilities induced by dissipation: it is well-known that internal viscous damping destabilizes the marginally stable Ziegler's pendulum and Pflüger column (of which the Beck's column is a special case), two structures loaded by a tangential follower force. The result is the so-called 'destabilization paradox', where the critical force for flutter instability decreases by an order of magnitude when the coefficient of internal damping becomes infinitesimally small. Until now external damping, such as that related to air drag, is believed to provide only a stabilizing effect, as one would intuitively expect. Contrary to this belief, it will be shown that the effect of external damping is qualitatively the same as the effect of internal damping, yielding a pronounced destabilization paradox. Previous results relative to destabilization by external damping of the Ziegler's and Pflüger's elastic structures are corrected in a definitive way leading to a new understanding of the destabilizating role played by viscous terms.

1. Introduction

1.1. A premise: the Ziegler destabilization paradox

In his pioneering work Ziegler (1952) considered asymptotic stability of a two-linked pendulum loaded by a tangential follower force P, as a function of the internal damping in the viscoelastic joints connecting the two rigid and weightless bars (both of length l, Fig. 1(c)). The pendulum carries two point masses: the mass m_1 at the central joint and the mass m_2 mounted at the loaded end of the pendulum. The follower force P is always aligned with the second bar of the pendulum, so that its work is non-zero along a closed path, which provides a canonical example of a non-conservative positional force.

For two non-equal masses ($m_1 = 2m_2$) and null damping, Ziegler found that the pendulum is marginally stable and all the eigenvalues of the 2×2 matrix governing the dynamics are purely imaginary and simple, if the load falls within the interval $0 \le P < P_u^-$, where

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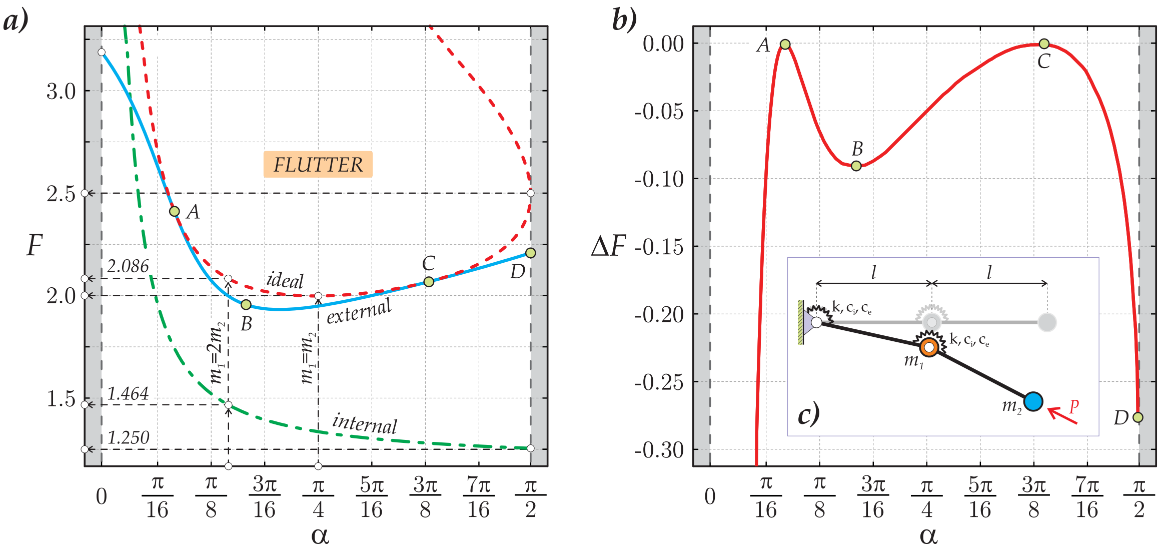

Fig. 1. (a) The (dimensionless) tangential force F, shown as a function of the mass ratio α (transformed via $\cot\alpha = m_1/m_2$), represents the flutter domain of (dashed/red line) the undamped, or 'ideal', Ziegler pendulum and the flutter boundary of the dissipative system in the limit of vanishing (dot-dashed/green line) internal and (continuous/blue line) external damping. (b) Discrepancy ΔF between the critical flutter load for the ideal Ziegler pendulum and for the same structure calculated in the limit of vanishing external damping. The discrepancy quantifies the Ziegler's paradox. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

$$P_u^- = \left(\frac{7}{2} - \sqrt{2}\right)\frac{k}{l} \approx 2.086\frac{k}{l}, \tag{1}$$

and k is the stiffness coefficient, equal for both joints. When the load P reaches the value P_u^-, two imaginary eigenvalues merge into a double one and the matrix governing dynamics becomes a Jordan block. With the further increase of P this double eigenvalue splits into two complex conjugate roots. The eigenvalue with the positive real part corresponds to a mode with an oscillating and exponentially growing amplitude, which is called flutter, or oscillatory, instability. Therefore, $P = P_u^-$ marks the onset of flutter in the *undamped* Ziegler's pendulum.

When the internal linear viscous damping in the joints is taken into account, Ziegler found another expression for the onset of flutter: $P = P_i$, where

$$P_i = \frac{41}{28}\frac{k}{l} + \frac{1}{2}\frac{c_i^2}{m_2 l^3}, \tag{2}$$

and c_i is the damping coefficient, assumed to be equal for both joints. The peculiarity of Eq. (2) is that in the limit of vanishing damping, $c_i \longrightarrow 0$, the flutter load P_i tends to the value $41/28\ k/l \approx 1.464\ k/l$, considerably lower than that calculated when damping is absent from the beginning, namely, the P_u^- given by Eq. (1). This is the so-called 'Ziegler's destabilization paradox' (Ziegler, 1952; Bolotin, 1963).

The reason for the paradox is the existence of the Whitney umbrella singularity on the boundary of the asymptotic stability domain of the dissipative system (Bottema, 1956; Krechetnikov and Marsden, 2007; Kirillov and Verhulst, 2010).[1]

In structural mechanics, two types of viscous dampings are considered: (i) one, called 'internal', is related to the viscosity of the structural material, and (ii) another one, called 'external', is connected to the presence of external actions, such as air drag resistance during oscillations. These two terms enter the equations of motion of an elastic rod as proportional respectively to the fourth spatial derivative of the velocity and to the velocity of the points of the elastic line.

Of the two dissipative terms only the internal viscous damping is believed to yield the Ziegler destabilization paradox (Bolotin, 1963; Bolotin and Zhinzher, 1969; Andreichikov and Yudovich, 1974).

1.2. A new, destabilizing role for external damping

Differently from internal damping, *the role of external damping is commonly believed to be a stabilizing factor*, in an analogy

[1] In the vicinity of this singularity, the boundary of the asymptotic stability domain is a ruled surface with a self-intersection, which corresponds to a set of marginally stable undamped systems. For a fixed damping distribution, the convergence to the vanishing damping case occurs along a ruler that meets the set of marginally stable undamped systems at a point located far from the undamped instability threshold, yielding the singular flutter onset limit for almost all damping distributions. Nevertheless, there exist particular damping distributions that, if fixed, allow for a smooth convergence to the flutter threshold of the undamped system in case of vanishing dissipation (Bottema, 1956; Bolotin, 1963; Banichuk et al., 1989; Kirillov and Verhulst, 2010; Kirillov, 2013).