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Macroscopic response of particle-reinforced elastomers subjected to prescribed torques or rotations on the particles

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ABSTRACT

Particle-reinforced rubbers are composite materials consisting of randomly distributed, stiff fibers/particles in a soft elastomeric material. Since the particles are stiff compared to the embedding rubber, their deformation can be ignored for all practical purposes. However, due to the softness of the rubber, they can undergo rigid body translations and rotations. Constitutive models accounting for the effect of such particle motions on the macroscopic response under prescribed deformations on the boundary have been developed recently. But, in some applications (e.g., magneto-active elastomers), the particles may experience additional torques as a consequence of an externally applied (magnetic) field, which, in turn, can affect the overall rotation of the particles in the rubber, and therefore also the macroscopic response of the composite. This paper is concerned with the development of constitutive models for particle-reinforced elastomers, which are designed to account for externally applied torques on the internally distributed particles, in addition to the externally applied deformation on the boundary of the composite. For this purpose, we propose a new variational framework involving suitably prescribed eigenstresses on the particles. For simplicity, the framework is applied to an elastomer reinforced by aligned, rigid, cylindrical fibers of elliptical cross section, which can undergo finite rotations in the context of a finite-deformation, plane strain problem for the composite. In particular, expressions are derived for the average in-plane rotation of the fibers as a function of the torques that are applied on them, both under vanishing and prescribed strain on the boundary. The results of this work will make possible the development of improved constitutive models for magneto-active elastomers, and other types of smart composite materials that are susceptible to externally applied torques.

1. Introduction

It has recently been proposed that the performance of magneto-active elastomers (MAEs) may be enhanced by the use of non-spherical particles (Ponte Castañeda and Galipeau, 2011; Galipeau and Ponte Castañeda, 2013). This is because the application of magnetic fields induces mechanical torques on non-spherical particles, which can be harnessed to produce large field-activated strains in MAEs consisting of aligned particles that are distributed randomly in an elastomer matrix. Indeed, the use of spherical particles induces only dipolar forces, which require close proximity of the particles—and therefore relatively large particle concentrations—to be effective in generating large forces. However, large particle

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concentrations also tend to increase the macroscopic stiffness of the composite, leading to the net result that the field-activated strains are relatively small in MAEs with spherical particles. On the other hand, non-spherical particles interact directly with the applied magnetic field and do not require the presence of other particles to experience torques. For this reason, they can be more effective in generating large field-activated strains, since large particle concentrations are not required. Completely analogous effects are present in dielectric elastomer composites (DECs) consisting of stiff dielectric particles in soft elastomer matrices, where the torques are generated by electrical fields instead of magnetic fields (Ponte Castañeda and Siboni, 2012; Siboni and Ponte Castañeda, 2014). Ignoring for the time being the source of the torques on the particles, this raises an interesting question in the theory of composite materials that seems to have been largely neglected thus far in the literature: How does the presence of externally produced torques on particles affect the macroscopic response of composite materials? In this work, we attempt to provide an answer to this question by generating estimates for the macroscopic response of particle-reinforced elastomers subjected to a combination of externally applied deformation on the outer boundary of the specimen and prescribed torques or rotations on the particles inside the composite. For simplicity, we will focus our attention here on the special case of fiber-reinforced elastomers with aligned fibers of elliptical cross section, but we consider finite strains and rotations. However, we should emphasize that the procedure is quite general and can be used for more general ellipsoidal particle shapes (e.g., Avazmohammadi and Ponte Castañeda, 2014a,b). As will be discussed further below in more detail, the solutions developed in this work for the effect of torques on particle-reinforced systems will allow the generation of improved estimates for MAEs and DECs in future work.

Although, to the best of the authors' knowledge, the above-mentioned problem has yet not been considered in the literature—even in the context of linear elasticity—it should be mentioned that a generalization of the single rigid inclusion problem (Eshelby, 1957) to account for additional particle rotations has been investigated by Walpole (1991). In addition, an application of this result to account for magnetic torques on a single ellipsoidal rigid particle embedded in an infinite linear-elastic medium has been considered recently by Siboni and Ponte Castañeda (2012). Finally, there are classical solutions (Rivlin, 1949; Kanner and Horgan, 2008) in the context of finite elasticity for cylindrical shells undergoing azimuthal shear that can be reinterpreted as solutions for the finite twist of single fibers of circular cross section in an elastomeric matrix.

In this paper we investigate the effect of externally applied torques or prescribed rotations on particles in composite materials, in the finite-deformation regime. This is, in general, a difficult task due to the material and geometric non-linearities associated with finite deformations, and for this reason we have to resort to approximate solutions. In particular, we will make use of the second-order linear comparison homogenization method of Ponte Castañeda (2002). This will allow us to obtain estimates for the effective energy function of composites with prescribed rotations (or torques) on the particles, as well as corresponding estimates for the relation between the resulting rotations and the externally applied torques. The resulting constitutive model accounts for simultaneously prescribed deformations on the boundary of the specimen, and thus reduces to the results of Lopez-Pamies and Ponte Castañeda (2006b) for the special case when the externally applied torques on the fibers vanish, so that the particles rotate freely as a consequence of the surrounding deformation in the elastomeric matrix phase of the composite.

The rest of this paper is organized as follows. In Section 2, we provide a brief overview of the homogenization problem for hyperelastic composites in the large deformation regime. Section 3 is concerned with the generalization of the homogenization procedure for cases where uniform eigenstresses are applied externally in each phase of the composite. This will be accomplished by making use of the fact that the effects of the applied eigenstresses can be accounted for by the addition of suitably chosen linear terms (in the deformation) to the phase energy functions. Section 4 is concerned with the second-order homogenization method. In particular, we consider the application of the generalized-secant, second-order method of Lopez-Pamies and Ponte Castañeda (2006a) for the modified variational problem with eigenstress, introduced in Section 3. Explicit expressions will be provided for both the effective energy function and for the average deformation of the phases in particulate composites. In Section 5, we focus our attention on two-phase composites with zero eigenstress in the matrix phase and non-zero eigenstress in the inclusion phase. We also provide specializations for two-phase composites consisting of rigid inclusions. Section 6 is concerned with the derivation of an explicit expression for the effective energy and the associated microstructure evolution for composites reinforced by long rigid fibers under 2D plane-strain conditions. In particular, explicit expressions are derived for the effective energy and average in-plane rotation of the fibers as a function of the prescribed eigenstress, which can then be related to the externally applied body couples (see Appendix A). In Section 7, we first compare the torque–rotation relation obtained in this paper with previously available results for single fibers of circular cross section. We then explore the effect of different microstructural variables, as well as the macroscopic deformation, on the torque–rotation response of fibers. We also investigate the effect of externally applied torques on the macroscopic constitutive response of the composite as determined by the relation between the effective Cauchy stress and the macroscopic deformation. Finally, in Section 8 we conclude the paper by discussing our findings and possible future directions.

2. Effective behavior

We consider a composite specimen, occupying the volume Ω_0 in the reference configuration, that consists of N different phases $\Omega_0^{(r)}$ with uniform but nonlinear stored-energy functions $W^{(r)}(\boldsymbol{F})$. As usual, $\boldsymbol{F}:=\mathrm{Grad}\chi(\mathbf{X})$ denotes the deformation gradient tensor of the map $\mathbf{x}=\chi(\mathbf{X})$ between the material points in the reference ($\mathbf{X}$) and deformed ($\mathbf{x}$) configurations. When