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Coupled hydro-mechanical effects in a poro-hyperelastic material

A.P.S. Selvadurai *, A.P. Suvorov

Department of Civil Engineering and Applied Mechanics, McGill University, 817 Sherbrooke Street West, Montréal, QC, Canada H3A 0C3

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ABSTRACT

Fluid-saturated materials are encountered in several areas of engineering and biological applications. Geologic media saturated with water, oil and gas and biological materials such as bone saturated with synovial fluid, soft tissues containing blood and plasma and synthetic materials impregnated with energy absorbing fluids are some examples. In many instances such materials can be examined quite successfully by appeal to classical theories of poroelasticity where the skeletal deformations can be modelled as linear elastic. In the case of soft biological tissues and even highly compressible organic geological materials, the porous skeleton can experience large strains and, unlike rubberlike materials, the fluid plays an important role in maintaining the large strain capability of the material. In some instances, the removal of the fluid can render the geological or biological material void of any hyperelastic effects. While the fluid component can be present at various scales and forms, a useful first approximation would be to treat the material as hyperelastic where the fabric can experience large strains consistent with a hyperelastic material and an independent scalar pressure describes the pore fluid response. The flow of fluid within the porous skeleton is defined by Darcy's law for an isotropic material, which is formulated in terms of the relative velocity between the pore fluid and the porous skeleton. It is assumed that the form of Darcy's law remains unchanged during the large strain behaviour. This approach basically extends Biot's theory of classical poroelasticity to include finite deformations. The developments are used to examine the poro-hyperelastic behaviour of certain one-dimensional problems.

1. Introduction

The mathematical theory of finite elasticity represents a defining development of modern non-linear continuum mechanics, a position achieved partly due to contributions of lasting value made by a number of individuals and partly due to its ability to provide meaningful and applicable results to many problems of technological interest. The history of the subject of finite elasticity can be traced back to the works of Cauchy, Green, Piola and others but the modern development of the subject commences with the seminal works of R.S. Rivlin, which have been compiled by Barenblatt and Joseph (1997). The contributions to the theory of elastic materials exhibiting large strain phenomena, subsequent to Rivlin's work, are far too numerous to be cited individually. Complete accounts of these developments are given in review and survey articles and volumes by Doyle and Ericksen (1956), Rivlin (1960), Adkins (1961), Spencer (1970), Beatty (1987) and in the volumes by

* Corresponding author.
E-mail address: patrick.selvadurai@mcgill.ca (A.P.S. Selvadurai).

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Murnaghan (1951), Eringen (1962), Green and Zerna (1968), Green and Adkins (1970), Wang and Truesdell (1973), Carlson and Shield (1980), Marsden and Hughes (1983), Ogden (1984), Hanyga (1985), Valent (1988), Lur'e (1990), Truesdell and Noll (1992), Antman (1995), Drozdov (1996), Carroll and Hayes (1996), Holzapfel (2000), Taber (2008) and Selvadurai (2015a). The pedagogical value of the theory of finite elasticity is highlighted in many excellent texts (Murnaghan, 1951; Eringen, 1962; Jaunzemis, 1967; Green and Zerna, 1968; Green and Adkins, 1970; Wang and Truesdell, 1973; Carlson and Shield, 1980; Marsden and Hughes, 1983; Ogden, 1984; Hanyga, 1985; Valent, 1988; Lur'e, 1990; Truesdell and Noll, 1992; Antman, 1995; Drozdov, 1996; Carroll and Hayes, 1996; Holzapfel, 2000; Taber, 2008) and in the compact volumes by Chadwick (1976), Gurtin (1981), Atkin and Fox (1980) and Spencer (2004). A survey of recent developments in the theory of non-linear elasticity is given by Beatty (2001a), Fu and Ogden (2001), Hill (2001) and the lecture series and volumes organized by Zubov (1997), Libai and Simmonds (1998), Dorfmann and Muhr (1999), Hayes and Saccomandi (2001), Saccomandi and Ogden (2004) and Dorfmann and Ogden (2004). The experimental aspects of hyperelastic materials date back to the work of Mooney (1940), Rivlin and Saunders (1951), Gent and Rivlin (1952a,b), Ogden (1972), Bell (1973) and Treloar (1975, 1976). Recent experimental investigations of hyperelastic rate-sensitive and non-rate sensitive materials are given by Pamplona and Bevilacqua (1992), Gent and Hua (2004), Selvadurai (2006) and Selvadurai and Shi (2012). The articles by Selvadurai (2006) and Selvadurai and Shi (2012) also provide extensive references to several other experimental investigations, notably those involving inflation and indentation of membranes, which have important applications in the identification of the constitutive behaviour of hyperelastic materials.

The study of the mechanics of fluid-saturated porous media has its origins in the modelling of consolidation of soils. A theory for the analysis of one-dimensional soil consolidation was first proposed by Terzaghi (1923). The model considers the linear deformability of the porous skeleton, Darcy flow of the saturating fluid through the pore space and an effective stress relationship between the stresses in the porous skeleton and the pore fluid pressures. The classical theory of poroelasticity proposed by Biot (1941) is a complete theory that accounts for three-dimensional elasticity and fluid transport effects in the formulation. The developments in the classical theory of Biot poroelasticity are quite extensive and these are documented in the review articles and volumes by Scheidegger (1960), Paria (1963), Rice and Cleary (1976), Schiffman (1984), Whitaker (1986), Coussy (1995), de Boer (2000), Wang (2000), Cowin (2001), Selvadurai (2007, 2015b), Verruijt (2015) and Cheng (2015). The classical theory has been extended by Selvadurai and Suvorov (2012, 2014) to include elasto-plastic behaviour of the porous skeleton, which constitutes an important aspect in the modelling of geomaterial behaviour. Applications of thermo-poroelasticity in the area of geosciences have also been discussed in a recent volume by Selvadurai and Suvorov (2016). The consideration of large strain effects in the modelling of saturated geomaterial behaviour is important to model the consolidation of soft sediments, although such materials can display irreversible phenomena in the constitutive behaviour of the porous skeleton. The influence of large strain phenomena on the one-dimensional consolidation of geomaterials was first examined by Gibson et al. (1967) who formulated the problem with reference to a Lagrangian approach for the description of the strains. The analysis takes into consideration the alterations in the void fraction in the porous skeleton, which in turn alters the permeability of the porous medium and its one-dimensional deformability properties. These alterations are also suggested by experimental evidence on saturated sediments. In the approach adopted by Gibson et al. (1967), the one-dimensional consolidation problem is reduced to a non-linear partial differential equation for the void ratio, which is converted to a linearized form and applied to examine consolidation problems of geotechnical interest. An extension of the classical theory of poroelasticity (Biot, 1941) to include finite deformations was also proposed by Biot (1972), although the developments were not adopted for the solution of problems in poro-hyperelasticity.

The finite strain one-dimensional soil consolidation problem has been extensively studied in the literature on soil mechanics and developments in this area are discussed by Gibson et al. (1981), Cargill (1984), Townsend and McVay (1990), Morris (2002, 2005), Ichikawa et al. (2010) and Ichikawa and Selvadurai (2012). The approach proposed in these studies is largely restricted to one-dimensional problems and suitable only for situations where there is no unloading of the consolidating soil; i.e. the deformability characteristics of the consolidating soil do not account for large strain irreversible processes in the porous skeleton. This can be addressed by implementing finite strain plasticity effects into the skeletal behaviour but such advances cannot be made solely through analytical approaches (Lee, 1969; Lubarda, 2001; Selvadurai and Yu, 2006a, 2006b, 2008; Yu and Selvadurai, 2007). The work of Uzuoka and Borja (2012) deals with the computational modelling of finite deformations of poro-hyperelastic behaviour of soils with a neo-Hookean form of a strain energy function, which again, does not account for irreversible deformations of the soil skeleton undergoing finite deformations.

Alternative perspectives of poromechanics have been considered in the context of the theory of mixtures by Green and Steel (1966), Crochet and Naghdi (1966), Mills and Steel (1970), Green and Naghdi (1970), Atkin and Craine (1976), Bowen (1976), Bedford and Drumheller (1983), Dell'Isola and Romano (1987), Coussy (1995), Murad et al. (1995), Rajagopal and Tao (1995), Bennethum and Cushman (1996), Drumheller (2000), Huyghe (2015a) and others. Of particular interest are the approaches presented by Shi et al. (1981), Dai et al. (1991), Baek and Srinivasa (2004), Gajo (2010) and Pence (2012). The article by Pence (2012) also contains a systematic treatment of the application of the mixture theory approach to fluid-saturated media and a comprehensive exposition of the current status. Also of particular interest to the discussion are the articles by Duda et al. (2010), Chester and Anand (2010) and Baek and Pence (2011) who examine the application of coupled theories in the context of polymer swelling. The mixture theory approaches are certainly elegant and complete from the point of view of continuum formulations, and are particularly relevant when a number of species saturating the porous space are encountered. The mixture theory-based formulations may have advantages when dealing with non-linear theories of material behaviour, where the porous skeleton can contain multi-species of pore fluids and the porous skeleton can