Journal of the Mechanics and Physics of Solids 84 (2015) 293–312

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

A theory of finite strain magneto-poromechanics

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ARTICLE INFO

Article history:
Received 30 March 2015
Received in revised form
19 July 2015
Accepted 6 August 2015
Available online 8 August 2015

Keywords:
Magneto-poromechanics
Continuum thermodynamics
Biot's theory
Large deformation
Magneto-active foams

ABSTRACT

The main purpose of this paper is the multi-physics modeling of magnetically sensitive porous materials. We develop for this a magneto-poromechanics formulation suitable for the description of such a coupling. More specifically, we show how the current state of the art in the mathematical modeling of magneto-mechanics can easily be integrated within the unified framework of continuum thermodynamics of open media, which is crucial in setting the convenient forms of the state laws to fully characterize the behavior of porous materials. Moreover, due to the soft nature of these materials in general, the formulation is directly developed within the finite strain range. In a next step, a modeling example is proposed and detailed for the particular case of magneto-active foams with reversible deformations. In particular, due to their potentially high change in porosity, a nonlinear porosity law recently proposed is used to correctly describe the fluid flow through the interconnected pores when the solid skeleton is finitely strained causing fluid release or reabsorption. From the numerical point of view, the variational formulation together with an algorithmic design is described for an easy implementation within the context of the finite element method. Finally, a set of numerical simulations is presented to illustrate the effectiveness of the proposed framework.

1. Introduction

Magneto-active polymers (MAPs) are mostly composites of a soft polymer matrix impregnated with magnetically permeable particles, typically iron particles in micro- or nano-meter size. In general, the response of MAPs to magnetic fields can be divided into two categories based on the property of the matrix material: they can give large and prompt deformation, or they can change their mechanical properties with moderate straining. These two features have received considerable attention in recent years due to their potential applications including, for instance, sensors, actuators, and biomedicine, see for example Jolly et al. (1996), Zrínyi et al. (1996), Ginder et al. (2002), Varga et al. (2006) among many others.

In parallel, the mathematical modeling of the coupling of electromagnetic fields in deformable materials has also been an area of active research. Fully coupled nonlinear field theories have been developed with constitutive formulations based on augmented free energy functions, see for instance Dorfmann and Ogden (2004a), Ericksen (2006), Kankanala and Triantafyllidis (2004), Steigmann (2004), Vu and Steinmann (2007). In particular, it has been shown that any one of the magnetic induction, magnetic field, or magnetization vectors can be used as an independent variable for the magnetic part of the problem, the other two being obtained through the constitutive relations. The relevant equations are based on the pioneering work of Pao (1978), see also Brown (1966), Kovetz (2000) for detailed discussions concerning these topics.

This work is devoted to the modeling of the particular case of magneto-active foams. These latter have a combination of

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http://dx.doi.org/10.1016/j.jmps.2015.08.003
0022-5096/

desirable properties, including high porosity, light weight, low cost and fast responsiveness to external stimuli. Indeed, they have the ability to respond to magnetic fields with drastic change in volume, shape, and porosity. Furthermore, when the porosity is highly interconnected, they can be good candidates for biomedical systems used to control drug delivery, see Liu et al. (2006), Zhao et al. (2011), or to dynamically control flows in microfluidic chips, see Hong et al. (2014).

It becomes then of interest to develop a theory that couples the magnetic field with the large deformation in porous media. Historically, two approaches have been used in a relevant literature for the modeling of porous materials: mixture theories, see for example Bowen (1982), Wilmanski (2003), and the macroscale consolidation theory of Biot, see for example Biot (1941, 1972). The former approach is mostly used to model species migration where the mixture equations for mass balance are used in combination with classical equations for linear momentum balance in terms of rule-of-mixture relations for the stress response, see the recent examples of application in Duda et al. (2010), Baek and Pence (2011) among others. The present work is based on the latter approach, i.e. Biot's theory. Since the pioneering work of Biot, considerable progress has been made in the last decades to develop a concise framework in the domain of poromechanics. Briefly, it describes the evolution of a saturated porous material in terms of the deformation of its solid skeleton part on the one hand, and in terms of the distribution of the mass of its fluid part, on the other hand. The resulting boundary value problem consists of a coupling between the balance equation and the mass conservation of the fluid. The reader is referred for example to Lewis and Schrefler (1998), Coussy (2004) for a detailed synthesis.

The coupling with magnetostatics is integrated within the framework of continuum thermodynamics of open media for the correct setting of the whole set of constitutive relations. In particular, to describe the potentially high change of porosity, we use a simplified version of the porosity law recently proposed in Nedjar (2013a), see also Nedjar (2013b). This law accounts for the physical property that the actual (Eulerian) porosity must belong to the interval $[0, 1]$ for any admissible process as, by definition, the porosity is at any time a ratio of the connected porous space. Among others, this allows for a good description of the seepage process and the fluid release and/or absorption during the loading history.

A further goal of this paper is the formulation of a finite element treatment to furnish a computational tool for structural simulations. The three-field boundary value problem at hand being strongly coupled, it must be solved with the help of a combination of existing numerical strategies proposed in a relevant literature. As a very first attempt, we opt for a monolithic scheme where the three sub-problems are solved simultaneously. The most relevant particularities of the proposed numerical scheme are highlighted for an easy implementation.

An outline of the remainder of this paper is as follows. In Section 2, we recall the governing equations of mass conservation and mechanical balance together with the specialized versions of Maxwell's equations. Both of the equivalent spatial and material descriptions are considered. Then, in Section 3, the magneto-mechanics coupling is embedded within the framework of continuum thermodynamics. In particular, we show how the formulation can be based on the magnetic induction vector or, equivalently, on the magnetic field vector. Section 4 is devoted to the modeling of hyperelastic magneto-active foams. Details of the whole constitutive equations are given together with the variational forms in view of the numerical approximation. This model example is then used for the simulations of Section 5. Finally, conclusions and perspectives are drawn in Section 6.

Notation: Throughout the paper, bold face characters refer to second- or fourth-order tensorial quantities. In particular, $\mathbf{1}$ denotes the second-order identity tensor with components δ_{ij} (δ_{ij} being the Kronecker delta), and $\boldsymbol{I}$ is the fourth-order unit tensor of components $I_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$. The notation $(\cdot)^T$ is used for the transpose operator and the double dot symbol ':' is used for double tensor contraction, i.e. for any second-order tensors $\boldsymbol{A}$ and $\boldsymbol{B}$, $\boldsymbol{A}:\boldsymbol{B} = \mathrm{tr}[\boldsymbol{A}\boldsymbol{B}^T] = A_{ij}B_{ij}$ where, unless specified, summation on repeated indices is always assumed. One has the property $\mathrm{tr}[(\cdot)] = (\cdot): \mathbf{1}$ for the trace operator "tr". The notation $\otimes$stands for the tensorial product. In components, one has $(\boldsymbol{A} \otimes \boldsymbol{B})_{ijkl} = A_{ij}B_{kl}$, and for any two vectors $\mathbb{U}$ and $\mathbb{V}$, $(\mathbb{U} \otimes \mathbb{V})_{ij} = U_iV_j$. Furthermore, the double-striked characters will exclusively be used for vector fields related to the magnetic part of the problem, e.g. $\mathbb{b}, \mathbb{B}, \mathbb{h}$ ….

2. Mass conservation and balance equations

When undeformed, unstressed, and in the absence of magnetic fields, the magnetically sensitive porous body occupies the reference configuration Ω_0 with boundary $\partial\Omega_0$. The porous body is thought as being a superimposition of a solid skeleton and a fluid phase. By solid skeleton, we mean the continuum formed from the constitutive matrix and the connected porous space emptied of fluid. Its deformation is the one that is observable under the combined action of mechanical forces and magnetic fields.

We identify a material solid skeleton particle by its position vector in the reference configuration, $\boldsymbol{X} \in \Omega_0$, and trace its motion by its current position at time t, $\boldsymbol{x}(\boldsymbol{X}, t) \in \Omega_t$. The deformation gradient is as usual defined as $\boldsymbol{F} = \nabla_{\boldsymbol{X}}\, \boldsymbol{x}$, where $\nabla_{\boldsymbol{X}}(\cdot)$ is the material gradient operator with respect to the reference coordinates $\boldsymbol{X}$. The Jacobian of the transformation is given by the determinant $J = \det \boldsymbol{F}$ with the standard convention $J > 0$.

Furthermore, for the porous space, we denote by n the Eulerian porosity which is the volume fraction of the connected porous space in the spatial configuration. Thus, for a current elementary volume $\mathrm{d}\Omega_t$ of porous material, the volume of porous space within it is $n\, \mathrm{d}\Omega_t$.

Now in contrast to the Eulerian porosity, the change in the porous space is thermodynamically better captured relative to the reference configuration through the Lagrangian porosity that we denote here by ϕ. This latter is defined by the following