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Transition from stick to slip in Hertzian contact with "Griffith" friction: The Cattaneo–Mindlin problem revisited

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ABSTRACT

Classically, the transition from stick to slip is modelled with Amonton–Coulomb law, leading to the Cattaneo–Mindlin problem, which is amenable to quite general solutions using the idea of superposing normal contact pressure distributions – in particular superposing the full sliding component of shear with a corrective distribution in the stick region. However, faults model in geophysics and recent high-speed measurements of the real contact area and the strain fields in dry (nominally flat) rough interfaces at macroscopic but laboratory scale, all suggest that the transition from 'static' to 'dynamic' friction can be described, rather than by Coulomb law, by classical fracture mechanics singular solutions of shear cracks. Here, we introduce an 'adhesive' model for friction in a Hertzian spherical contact, maintaining the Hertzian solution for the normal pressures, but where the inception of slip is given by a Griffith condition. In the slip region, the standard Coulomb law continues to hold. This leads to a very simple solution for the Cattaneo–Mindlin problem, in which the "corrective" solution in the stick area is in fact similar to the mode II equivalent of a JKR singular solution for adhesive contact. The model departs from the standard Cattaneo–Mindlin solution, showing an increased size of the stick zone relative to the contact area, and a sudden transition to slip when the stick region reaches a critical size (the equivalent of the pull-off contact size of the JKR solution). The apparent static friction coefficient before sliding can be much higher than the sliding friction coefficient and, for a given friction fracture "energy", the process results in size and normal load dependence of the apparent static friction coefficient. Some qualitative agreement with Fineberg's group experiments for friction exists, namely the stick–slip boundary quasi-static prediction may correspond to the arrest of their slip "precursors", and the rapid collapse to global sliding when the precursors arrest front has reached about half the interface may correspond to the reach of the "critical" size for the stick zone.

1. Introduction

The basic laws of (dry) friction are simple and have been understood centuries ago, and independence of the frictional coefficient from the macroscopic shape of the body and the normal load has been justified by various models, invoking plasticity (Bowden and Tabor, 1950), or more recently, multiscale and statistical arrangement of asperities (Archard, 1957; Greenwood and Williamson, 1966), which all lead to the linearity of tangential load with normal load (Coulomb law), even

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in the presence of non-linearities at the asperity scale (Barber, 2013).

Recently, an interesting series of experiments has been conducted by the group of Jay Fineberg in Israel (Rubinstein et al., 2007; Ben-David et al., 2010a, 2010b; Ben-David and Fineberg, 2011; Svetlizky and Fineberg, 2014). In their experiments, they use two blocks of transparent material (PMMA, poly methyl-methacrylate), which permits real-time visualization of the net contact area that forms the interface separating two blocks of like material. Sliding is seen indirectly as a "change of contact area" and a complex pattern of propagating fronts at the interface exists, with both classical fast fronts, and less known slower fronts, which were found to be the dominant mechanism for the rupture of the interface: the slow fronts introduce the typical instabilities and kinks in the global force–displacement curve and arrest at progressively longer distance from the leading edge where slip initiates. Although viscoelastic and dynamic effects limit the applicability of quasi-static models, no attempt has been made so far (that we know of) of quasi-static predictions for the problem, despite clearly the loading regime is well within quasi-static limits, and here we shall try to commence some discussion in this direction. The setup was increasingly improved measuring with some accuracy the shear and normal stresses along the interface (Ben-David et al., 2010a, 2010b), finding that their ratio can locally far exceed (a ratio of about 4–6) the static friction coefficient without precipitating slip — this strongly suggests a difference between friction defined at the microscopic scale and the macroscopic friction. More recently, it has been shown that the strain fields compare well with classical fracture mechanics singular solutions of shear cracks (Svetlizky and Fineberg, 2014), at all speeds, except perhaps the very fast propagation. The critical energy for propagation (or a critical mode II stress intensity factor, friction "toughness") is nearly constant, or perhaps depends only on the local pressure, via the real contact area reduction of the nominal interface area. Further, it is of the order of 1/1000 of the bulk value (2000 J/m^2), but this (1 J/m^2) is still 1–2 orders of magnitude higher than surface energy responsible of adhesion (40–50 mJ/m^2) for PMMA, so although we are introducing a fracture mechanics model for friction, this has nothing to do with adhesion at the interface in the classical sense, which we can assume destroyed by roughness, as well known from Fuller and Tabor model (1975).

Ida (1972) and Palmer and Rice (1973) perhaps firstly introduced fracture mechanics concept in friction, also to explain some failures when the shear stress was everywhere below the expected limiting value. Studies of natural faults often recur to fracture mechanics (Scholz, 2002; Brener and Marchenko, 2002; Ben-Zion, 2008), where frictional size effects are well known (Carpinteri and Paggi, 2008; Paggi and Carpinteri, 2013), although experimental evidence may have been obtained firstly by Svetlizky and Fineberg (2014). Pure mode II occurs hardly in materials, because the cracks are more prone to rotate into mode I failure. At the other extreme of the spectrum in size, single adhesive contacts have been studied with Fracture Mechanics both for the extent of the contact area (JKR theory for Hertzian spheres), and for the interaction of friction with adhesion (Johnson, 1997; Adams, 2014) or to explain size-scale effects (Hurtado and Kim, 1999a, 1999b). It should be immediately borne in mind that we depart from these nano-micro models, in many respects (we do not consider a mode I adhesion occurring at the contact edge, or slip occurring at a constant shear stress, for example), as we are looking at macroscopic sizes.

Turning back to the classical solutions using Coulomb–Amontons' law of friction, Cattaneo (1938) and Mindlin (1949) solved the quasi-static transition from stick to slip for a Hertzian geometry (i.e. with initial gap described by quadratic functions), where the pressures are unaffected tangential displacements (which requires elastically similarity of the two contacting materials). They showed that complete stick needs an infinite shear stress at the boundary of the contact and this violated Coulomb's law without adhesion. Hence, they found the shear tractions by a clever superposition of the full sliding term, with a corrective term which they found ad hoc to impose the condition of constant tangential displacements in the stick zone. Later on, this procedure was generalized for arbitrary geometries (Ciavarella, 1998a, 1998b) showing that the corrective term is always proportional to the pressure distribution when the contact area is the stick zone, and also to harmonic loading, with a more complicated superposition (Putignano et al., 2011).

The present model assumes that the slip zones are governed by Coulomb's law so that the full sliding regime returns to the standard Coulomb behavior,[1] while in the stick zone a singular shear stress can occur, and infinite local "coefficient of friction" can occur. The resulting Cattaneo–Mindlin is solved by superposition and in particular, the JKR solution is proved to be the basis for the singular shear stress. The extent of the stick zone, the possible increase of friction force before sliding, and the full solution are obtained. Implications are discussed.

2. The model

In the contact between two elastic bodies, it is often the case that the two materials are identical or elastically similar, that is to say Dundurs' elastic constant β is zero (Johnson, 1985) so that the pressure distribution does not affect shear tractions distributions and viceversa. Further, it is customary to assume that, loading in a given tangential direction x only shear tractions oriented along that direction are generated: this corresponds to neglect Poisson's ratio effect which has however proved to be very small (see Ciavarella, 1998b; Munisamy et al., 1994; Putignano et al., 2011), since the self-equilibrated distribution of shear traction in the other direction does not lead to significant effects in terms of global

[1] Contrary to Adams–Johnson's models, which have a constant friction force proportional to the contact area and hence non-linearly to the normal load via JKR.