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On the equivalence between traction- and stress-based approaches for the modeling of localized failure in solids

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ABSTRACT

This work investigates systematically traction- and stress-based approaches for the modeling of strong and regularized discontinuities induced by localized failure in solids. Two complementary methodologies, i.e., *discontinuities localized in an elastic solid* and *strain localization of an inelastic softening solid*, are addressed. In the former it is assumed *a priori* that the discontinuity forms with a continuous stress field and along the known orientation. A traction-based failure criterion is introduced to characterize the discontinuity and the orientation is determined from Mohr's maximization postulate. If the displacement jumps are retained as independent variables, the strong/regularized discontinuity approaches follow, requiring constitutive models for both the bulk and discontinuity. Elimination of the displacement jumps at the material point level results in the embedded/smeared discontinuity approaches in which an overall inelastic constitutive model fulfilling the static constraint suffices. The second methodology is then adopted to check whether the assumed strain localization can occur and identify its consequences on the resulting approaches. The kinematic constraint guaranteeing stress boundedness and continuity upon strain localization is established for general inelastic softening solids. Application to a unified stress-based elastoplastic damage model naturally yields all the ingredients of a localized model for the discontinuity (band), justifying the first methodology. Two dual but not necessarily equivalent approaches, i.e., the traction-based elastoplastic damage model and the stress-based projected discontinuity model, are identified. The former is equivalent to the embedded and smeared discontinuity approaches, whereas in the later the discontinuity orientation and associated failure criterion are determined consistently from the kinematic constraint rather than given *a priori*. The *bi-directional* connections and equivalence conditions between the traction- and stress-based approaches are classified. Closed-form results under plane stress condition are also given. A generic failure criterion of either elliptic, parabolic or hyperbolic type is analyzed in a unified manner, with the classical von Mises (J_2), Drucker–Prager, Mohr–Coulomb and many other frequently employed criteria recovered as its particular cases.

1. Introduction

Overall responses of inelastic softening solids are characterized by strain localization, i.e. a manifestation of concentration of micro-structural defects. Depending on the material of interest, the phenomena resulting from strain

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localization may be diverse: dislocations of order of microns in crystal metals, cracks of order of millimeters in concrete, and shear bands of order ranging from millimeters to kilometers in granular and geological problems. From the structural point of view these localization band may be regarded as a fracture surface of small or even negligible width compared to the length scale of the structure. Structural collapse is often induced by formation of such localized failure. Therefore, it is of utmost significance to evaluate (residual) structural safety once strain localization occurs, and to prevent potential catastrophic collapse caused by localized failure. However, despite the recent progresses made, the modeling of strain localization and subsequent structural collapse still remains a challenging issue.

Strain localization inevitably induces strain/displacement discontinuities, hindering the applicability of classical continuum mechanics. With respect to the strategies for the approximation of such discontinuities and resulting consequences on material or structural responses, different approaches have been proposed ever since the pioneering work of Ngo and Scordelis (1967) and Rashid (1968). Generally speaking, existing approaches can be classified into the nonlinear fracture mechanics based discontinuous models or the generalized continuum mechanics based continuous models. In the computational context, they correspond to the discrete and smeared discontinuity methods, respectively.

In the discontinuous (discrete) approach strain/displacement jumps are explicitly accounted for by embedding the discontinuities into a solid matrix along preferred orientations. It is generally assumed that energy dissipation mechanisms are localized into the discontinuities while the bulk remains elastic. The traction continuity condition is imposed between them. The overall inelastic behavior is of anisotropy by construction. To characterize the dissipative behavior lumped in the discontinuities, vectorial traction-based cohesive models furnished with the fracture energy are introduced. Generally, displacement discontinuities are regarded as zero-width failure surfaces characterized by tractions vs. displacement jumps (Barenblatt, 1959, 1962; Dugdale, 1960; Hillerborg et al., 1976). Alternatively, strain discontinuities across the localized band with a finite width can be represented in terms of tractions vs. inelastic deformations (i.e., apparent displacement jumps normalized with respect to the bandwidth) (Cervera, 2008a,b). Depending on the recoverable/irreversible properties of the discontinuities, traction-based cohesive models of either plastic (Stankowski et al., 1993; Carol et al., 1997; Weihe et al., 1997), damage (Armero, 1999; Armero and Oller, 2000; Jirásek and Zimmermann, 2001) or combined plastic–damage (Wu and Xu, 2011; Wu, 2011) type can be established.

Contrariwise, the continuous approach relies on the introduction of a generalized continuum, so that stress-based constitutive models with regularized softening regime can be used. In this approach, the strain/displacement discontinuities are regularized (smeared) in such a way that the classical concepts of (average) stress and strain still apply. It is no longer necessary to make distinction between the elastic bulk and the inelastic discontinuity (band). But rather, the overall nonlinear behavior of the weakened medium is described by generalized constitutive laws in terms of stress vs. strain tensors equipped with softening internal variables. In this way, plasticity and damage mechanics or their combination (Chen, 1994; Krajcinovic, 2003) can be employed to develop appropriate inelastic constitutive laws. Discontinuity induced anisotropy can be considered either theoretically (Meschke et al., 1998; Govindjee et al., 1995; Wu and Xu, 2011) or in the computational context (Cervera and Chiumenti, 2006a, 2006b). Furthermore, to guarantee objectivity of the energy dissipated during the fracture process, the softening regime has to be regularized with respect to the length scale of localization band. The fracture energy, a material property measuring the dissipation per fracture surface area, and an appropriately identified localization bandwidth (Bažant and Oh, 1983; Oliver, 1989) are fundamental ingredients for this purpose.

In the traction-based approach for the modeling of localized failure in solids, a crucial step is to determine the discontinuity orientation. This is a non-trivial goal for a new or propagating discontinuity whose orientation is not pre-defined or known *a priori*. To this end, *ad hoc* strategies have to be introduced, usually in a heuristic manner. For instance, the maximum tensile stress (i.e., Rankine) criterion is often adopted for mode I failure in quasi-brittle materials. For more general cases, selecting the discontinuity orientation according to some makeshift condition and fixing it afterwards becomes a superimposed condition on the material behavior. Fortunately, the occurrence of necessary requisites for a certain type of failure to be initiated during the whole deformation process in inelastic solids provides useful information. In this aspect, the pioneering works by Hill (1958, 1962), Thomas (1961) and Rice (Rudnicki and Rice, 1975; Rice and Rudnicki, 1980) have been widely adopted in the literature. The sufficient and necessary conditions for discontinuous bifurcation and strain localization of elastoplastic materials were identified (Borré and Maier, 1989) and formulations for the orientation of shear bands were obtained; see Runesson et al. (1991) and the references therein. Discontinuous bifurcation analysis, based on the assumption of linear comparison solid and the traction continuity condition, has now become the standard tool to analyze propagation of strain or weak discontinuities.

For strong discontinuities in solids with strain softening regimes, there is no consensus for the determination of the discontinuity orientation. For instance, Simó et al. (1993) and Oliver (1996) suggested using the discontinuous bifurcation condition with perfect (null) softening/hardening modulus to determine the discontinuity orientation. However, this condition in general does not guarantee the occurrence of strong discontinuities (Oliver et al., 1998, 1999). To remedy this problem, a variable bandwidth model was proposed in the last reference, so that weak discontinuities can evolve smoothly to a strong one at a later stage. However, for strain localization to occur in a softening solid and develop eventually into a fully softened discontinuity at the final stage of the deformation process, material points inside the discontinuity (band) undergo inelastic loading while those outside it unload elastically. Therefore, provided the strong discontinuity cannot be guaranteed, the limit decohesion is not achieved in general cases and stress locking occurs due to the mis-prediction of the discontinuity orientation. Furthermore, due to the singular strain field associated with displacement discontinuities, traction continuity alone is not sufficient to guarantee physically meaningful results, but rather, stress boundedness also has to be