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Forerunning mode transition in a continuous waveguide

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ABSTRACT

We have discovered a forerunning mode transition as the periodic wave changing the state of a uniform continuous waveguide. The latter is represented by an elastic beam initially rested on an elastic foundation. Under the action of an incident sinusoidal wave, the separation from the foundation occurs propagating in the form of a *transition wave*. The critical displacement is the separation criterion. Under these conditions, the steady-state mode exists with the transition wave speed independent of the incident wave amplitude. We show that such a regime exists only in a bounded domain of the incident wave parameters. Outside this domain, for higher amplitudes, the steady-state mode is replaced by a set of local separation segments periodically emerging at a distance ahead of the main transition point. The crucial feature of this waveguide is that the incident wave group speed is greater than the phase speed. This allows the incident wave to deliver the energy required for the separation. The analytical solution allows us to show in detail how the steady-state mode transforms into the forerunning one. The latter studied numerically turns out to be periodic. As the incident wave amplitude grows the period decreases, while the transition wave speed averaged over the period increases to the group velocity of the wave. As an important part of the analysis, the complete set of solutions is presented for the waves excited by the oscillating or/and moving force acting on the free beam. In particular, an asymptotic solution is evaluated for the resonant wave corresponding to a certain relation between the load's speed and frequency.

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1. Introduction

An unusual *transition wave* considered in this paper belongs to the class of processes like phase transition or crack growth (which also can be considered as a phase transition (Truskinovsky, 1996)), or other similar events, where a change of the body structure or state spreads as a wave. The transition requires a certain energy, and such a wave can propagate being forced by the action of external loads or spontaneously drawing energy stored in the waveguide (Mishuris and Slepyan, 2014; Ayzenberg-Stepanenko et al., 2014). The transition wave in an elastic beam initially rested on a continuous or a discrete elastic foundation (Brun et al., 2013) can be considered as an example. In this work, a (negative) jump in the foundation stiffness propagates under gravity forces as a steady-state wave. (This was a theoretical base related to a bridge collapse, Brun et al., 2014.)

In this paper, the separation from the foundation under a sinusoidal incident wave is considered. Two ordered regimes of the transition wave are found: steady-state and forerunning. Below, in the Introduction, the main features of such processes

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are outlined and then the physical formulation of the considered problem is given.

The formulation of such a problem includes the dynamic equations for the waveguide in its initial and final states and, in addition, relations for the transition from the former to the latter. Note that the transition can occur instantaneously or during a period. The former mode of transition adopted in this paper is simpler for mathematical analysis. In this case, the waveguide is separated by a moving transition point (or an interface) into two parts, the intact part is placed in front of this point, while the modified part is placed behind this point. Note that generally, in the framework of a continuous waveguide, the formulation of the transition criterion is not trivial (Slepyan, 2002).

In analytical studies, transition waves are commonly examined under the steady-state formulation, assuming that the dynamic state depends on $\eta = x - Vt$ but not on x and t separately (x is the coordinate, t is time and V is the transition wave speed). The steady-state solution is based on the above-mentioned equations, the external load or/and conditions at infinity and some local conditions concerning the transition: a critical displacement, force or an energy release. An additional inequality, also local, can be needed in the case of a wave action. These equations and conditions can define the solution uniquely. Alternatively, a set of solutions is defined by such a formulation. In this latter case, some nonlocal conditions should be involved.

Mathematically, these considerations are sufficient. However, from the physical point of view it is not so. The main question is whether the steady-state regime can exist. This question arises due to the fact that the solution must satisfy certain additional, nonlocal conditions concerning the waveguide state at $\eta \neq 0$.

This may be a kinematic condition behind the transition point, $\eta < 0$, which can restrict the displacements, and a condition in front of the transition point, $\eta > 0$, which restricts a state parameter level to be subcritical (in accordance with the formulation, it becomes critical only at $\eta = 0$).

The latter condition is crucial for the problem under consideration. It is known as the *admissibility condition* (Marder and Gross, 1995), which states, in general, that the transition criterion should not be satisfied at $\eta > 0$, that is before the moment assumed in the problem formulation. Note that this condition was formulated for the lattice fracture; however, it is valid in a general case. This condition allows one to select a consistent solution from the set defined by the steady-state formulation. It also can essentially bound the region of existence of the steady-state regime.

As an example, the first analytical work on the lattice fracture (Slepyan, 1981a) can be mentioned. In this paper, the macrolevel energy release rate is obtained as a function of two parameters, the microlevel fracture energy and the crack speed. The corresponding micro-to-macro energy ratio is obtained as a non-monotonic function of the speed, such that there are some different crack speeds corresponding to a given macrolevel energy release rate. The Marder–Gross condition results in the conclusion that only the highest crack speed is admissible (if no fracture occurs outside of the prospective crack line).

In the context of the considered problem, another point is important. There is an essential difference in steady-state transition excited by a non-oscillating and oscillating incident waves. Under the action of a non-oscillating wave, the speed of the transition point increases approaching the incident wave group velocity as the action of the force increases. In contrast, in the steady-state transition under a sinusoidal incident wave, the speed of the transition point coincides with the phase speed of the incident wave regardless of the wave amplitude. In this case, as long as an energy release is required for the transition, the incident wave group speed must exceed the phase speed (since the energy flux velocity is equal to the group velocity). It can be seen below that under a sufficiently large wave amplitude, the transition point speed is below the group speed but faster than the phase speed of the incident wave. However, this appears in a different transition wave mode: it is not steady-state any longer.

The analytical solutions for fracture under a sinusoidal wave were presented by Slepyan (1981b, 2010) for a lattice and a continuous body, respectively. The papers most related to the considered issue, how the steady-state regime is replaced by a more complicated ordered mode, relate to the lattice fracture dynamics, Mishuris et al. (2009) and Slepyan et al. (2010). It was first found and discussed in these papers that there exist ordered clustering regimes changing as the incident wave amplitude grows. In a bounded wave amplitude region, the crack speed is equal to the incident wave phase speed, and the steady-state regime is valid. In this regime, the lattice bonds on the crack path break one after another at regular time intervals. Then, as the wave amplitude exceeds the critical level, the two-bond clustering occurs with two alternating values of the local crack speed. The crack speed averaged over the cluster again is constant but greater than the phase speed of the incident wave. In the further growth of the wave amplitude, the number of the bonds in the cluster increases, while the averaged crack speed is constant in each corresponding wave amplitude region. As the wave amplitude grows this averaged-over-the-cluster crack speed approaches the group speed of the incident wave. Such a clustering was also observed in the spontaneous crack propagation in a two-line chain with internal potential energy, Ayzenberg-Stepanenko et al. (2014). Note that the transition waves in lattices were considered in many works, see, e.g., Slepyan and Ayzenberg-Stepanenko (2004), Slepyan et al. (2005), Vainchtein (2010) and the references herein.

In the present work, we consider a beam initially rested on an elastic foundation and subjected to the action of a sinusoidal wave. Under this action, the separation of the beam from the foundation propagates as a transition wave. The aim is to find, in such a continuous waveguide, the domain where the steady-state regime can exist and to study the nontrivial transition mode existing outside this domain. In this simple model, the transformation of the steady-state mode into more complicated forerunning mode can be observed in detail. The ordered forerunning mode manifests itself as a periodic process as in the case of the above-mentioned clustering mode in the lattice; however, the forerunning mode differs much from the latter. The main peculiarity is the periodic occurrence of detachment segments in front of the main transition wave

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