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Modelling the torsion of thin metal wires by distortion gradient plasticity

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ABSTRACT

Under small strains and rotations, we apply a phenomenological higher-order theory of *distortion* gradient plasticity to the torsion problem, here assumed as a paradigmatic benchmark of small-scale plasticity. Peculiar of the studied theory, proposed about ten years ago by Morton E. Gurtin, is the constitutive inclusion of the *plastic spin*, affecting both the free energy and the dissipation. In particular, the part of the free energy, called the defect energy, which accounts for Geometrically Necessary Dislocations, is a function of Nye's dislocation density tensor, dependent on the plastic distortion, including the plastic spin. For the specific torsion problem, we implement this distortion gradient plasticity theory into a Finite Element (FE) code characterised by implicit (Backward Euler) time integration, numerically robust and accurate for both viscoplastic and rate-independent material responses. We show that, contrariwise to other higher-order theories of *strain* gradient plasticity (neglecting the plastic spin), the distortion gradient plasticity can predict some strengthening even if a quadratic defect energy is chosen. On the basis of the results of many FE analyses, concerned with (i) cyclic loading, (ii) switch in the higher-order boundary conditions during monotonic plastic loading, (iii) the use of non-quadratic defect energies, and (iv) the prediction of experimental data, we mainly show that (a) including the plastic spin contribution in a gradient plasticity theory is highly recommendable to model small-scale plasticity, (b) less-than-quadratic defect energies may help in describing the experimental results, but they may lead to anomalous cyclic behaviour, and (c) dissipative (unrecoverable) higher-order *finite* stresses are responsible for an unexpected mechanical response under non-proportional loading.

1. Introduction

The torsion of thin metal wires is one of the few paradigmatic examples of the small-scale plasticity behaviour involving the size effect nowadays identified by the expression “smaller being stronger”, as firstly shown by the experimental results of Fleck et al. (1994). Among other experiments providing evidence of such size effect, due to inhomogeneous plastic flow on the scale of several micrometers and below, let us mention the bending of thin foils investigated by Stölken and Evans (1998).

In the torsion experiments on *polycrystalline* copper wires, Fleck et al. (1994) observed, with diminishing specimen size,

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both an increase in strain hardening and a conspicuous strengthening, the latter being an increase of what is recognised, in a properly normalised monotonic torque–twist curve, as the torque value at the abrupt change of slope between the elastic and "fully plastic" regimes. Moreover, Fleck et al. (1994) have experimentally shown that the size effect depends on the loading conditions, observing that the uniaxial tensile response (of the same type of specimens used for the torsion tests) exhibits almost negligible size effect. In the specimens tested by Fleck et al. (1994) the diameter ranges between 12 and 170 μm and the grain size between 5 and 25 μm, with larger wires having larger grains. The Hall–Petch size effect should be, in this case, not much relevant, while the main reason for the observed size effect should be related to the imposed gradient of deformation, as postulated in the pivotal work of Ashby (1970) on the relation between Geometrically Necessary Dislocations (GNDs) and plastic strain gradients. Experimental results qualitatively similar to those of Fleck et al. (1994) have been recently obtained by Liu et al. (2013), still on polycrystalline copper wires.

We are concerned with the modelling of the torsion of thin metal wires by the phenomenological gradient plasticity (GP) theory proposed by Gurtin (2004). The theory is said to be "phenomenological" (or "isotropic") for it neglects any characterisation of the crystal lattice, contrariwise to "crystal" theories.

The peculiarity which distinguishes Gurtin's (2004) gradient theory from most of the other phenomenological GP theories available in the literature is the constitutive inclusion of the plastic spin, that is the skew-symmetric part of the plastic distortion. Gurtin's theory (Gurtin, 2004) accounts for the dissipation due to the plastic spin together with its energetic (or recoverable) higher-order counterpart included in the free energy. Such a higher-order addition to the free energy consists of a function, called the *defect energy*, of Nye's dislocation density tensor $\boldsymbol{\alpha}$ (Nye, 1953; Fleck and Hutchinson, 1997; Arsenlis and Parks, 1999), that is the curl of the plastic distortion $\boldsymbol{\gamma}$. In this paper, in order to distinguish the phenomenological GP theory here employed from the more common *strain* gradient plasticity (SGP) phenomenological theories, that are those overlooking the contribution of the plastic spin, we call the former *distortion* gradient plasticity (DGP) theory. After the seminal work of Aifantis (1984, 1987), noticeable examples of SGP theories have been developed by Fleck et al. (1994, 2014), Fleck and Hutchinson (1997, 2001), Niordson and Hutchinson (2003), Forest and Sievert (2003), Gudmundson (2004), Gurtin and Anand (2005, 2009), Polizzotto (2007, 2010), Fleck and Willis (2009a,b), Kuroda and Tvergaard (2010), and Hutchinson (2012). All these gradient theories rely on the definition of higher-order (i.e., unconventional) stresses work-conjugate to some appropriate measure of strain (or distortion) gradient. This spontaneously leads to higher-order boundary conditions, that are useful to describe the behaviour of dislocation pile-ups, this behaviour being one of the main sources of the observed size effects.

We consider both energetic and dissipative (that is, unrecoverable) higher-order stresses, as common in the most advanced phenomenological SGP theories (see, e.g., Gudmundson, 2004; Gurtin and Anand, 2005; Fleck and Willis, 2009b, and references therein). In particular, we consider the DGP theory introduced in section 12 of Gurtin (2004), in which a higher-order unrecoverable stress is work-conjugate to the gradient of the plastic strain rate. For dimensional consistency this leads to the introduction of a "dissipative" material length scale, of different nature with respect to the "energetic" material length scale(s) entering, still for dimensional consistency, the defect energy.

The DGP theory of Gurtin (2004) has been studied in Bardella (2009, 2010), where it is shown that the contribution of the plastic spin plays a fundamental role (even for small displacements and rotations) in order to best describe, in the phenomenological GP context, the mechanical response predicted by strain gradient *crystal* plasticity, the latter being more suitable to model small-scale plasticity. In particular, Bardella (2009), on the basis of a crystal GP model and Γ-convergence results obtained by Bardella and Giacomini (2008), could identify the material parameter χ governing the dissipation due to the plastic spin in Gurtin's model, in terms of other material parameters and length scales involved in the modelling. Then, Bardella (2010) has shown that the findings of Bardella and Giacomini (2008) and Bardella (2009), analytically established for quadratic dissipation potential, about the influence of material parameters and length scales, apply also to dissipation potentials suitable for metal plasticity (that are well described by a one-homogeneous function of the effective plastic flow rate). The studies (Bardella and Giacomini, 2008; Bardella, 2009, 2010) are limited to the simple shear of a strip constrained between two regions impenetrable to dislocations, and the results are obtained in the deformation theory context. In this work, we further investigate on Gurtin's (2004) DGP flow theory by applying it to the torsion problem of polycrystalline thin wires, in which the plastic spin should give an important contribution. Moreover, in torsion, multislip may more likely occur, suggesting that the mechanical response might be close to the ideal one analytically studied in Bardella and Giacomini (2008) and Bardella (2009), where the crystal model is made isotropic by assuming that any direction be an active slip system.

Recently, Poh (2013) has employed Gurtin (2004) theory to phenomenologically describe the local crystal behaviour in a homogenisation procedure for polycrystals. Formerly, Ostien and Garikipati (2008) proposed a discontinuous Galerkin Finite Element (FE) implementation of Gurtin (2004) flow theory for the analysis of two-dimensional boundary value problems. Ebobisse and Neff (2010) and Nesenenko and Neff (2012, 2013) have demonstrated the well-posedness and other mathematical properties of a DGP which neglects dissipative (unrecoverable) higher-order stresses. Polizzotto (2010) has developed a rate-independent DGP theory in which the free energy includes a quadratic function of plastic strain, plastic strain gradient, and plastic spin gradient. Also Berdichevsky (2006) has pointed out the importance of accounting for the plastic spin in modelling small-scale plasticity.

The torsion of thin wires has been modelled by using different SGP theories by Fleck et al. (1994), Gao et al. (1999), Huang et al. (2000), Gudmundson (2004), Borino and Polizzotto (2007), Idiart and Fleck (2010) (who have applied the Fleck and Willis, 2009b SGP), Chiricotto et al. (2012) (who have applied the Gurtin and Anand, 2005 SGP), and Liu et al. (2013).