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Modelling volumetric growth in a thick walled fibre reinforced artery

T.S.E. Eriksson [a,*], P.N. Watton [b,c], X.Y. Luo [d], Y. Ventikos [e]

[a] *Department of Engineering Science, Institute of Biomedical Engineering, University of Oxford, UK*
[b] *Department of Computer Science, University of Sheffield, UK*
[c] *INSIGNEO Institute for in silico Medicine, University of Sheffield, UK*
[d] *School of Mathematics and Statistics, University of Glasgow, UK*
[e] *Department of Mechanical Engineering, University College London, UK*

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ABSTRACT

A novel framework for simulating growth and remodelling (G&R) of a fibre-reinforced artery, including volumetric adaption, is proposed. We show how to implement this model into a finite element framework and propose and examine two underlying assumptions for modelling growth, namely *constant individual density* (CID) or *adaptive individual density* (AID). Moreover, we formulate a novel approach which utilises a combination of both AID and CID to simulate volumetric G&R for a tissue composed of several different constituents. We consider a special case of the G&R of an artery subjected to prescribed elastin degradation and we theorise on the assumptions and suitability of CID, AID and the mixed approach for modelling arterial biology. For simulating the volumetric changes that occur during aneurysm enlargement, we observe that it is advantageous to describe the growth of collagen using CID whilst it is preferable to model the atrophy of elastin using AID.

1. Introduction

The focus of this paper is to develop concepts for simulating the volumetric changes that occur to a fibre reinforced composite soft-tissue such as arteries as a consequence of growth and remodelling (G&R) of the constituents. Most prior works on arterial G&R utilised either conceptual geometries, membrane formulations or simplified axisymmetric motions, see, e.g., the works by Gleason and Humphrey (2005), Baek et al. (2006), Eriksson et al. (2009), Watton and Hill (2009), Watton et al. (2011a), Valentín et al. (2011) to name a few. These works have all contributed to set the foundation, and provide novel insights for arterial G&R. However, in a fairly recent review article by Humphrey and Holzapfel (2012), the need for more advanced patient-specific computational models is discussed. For these types of models to become reality, there is a need to handle arbitrary geometries and thick-walled and volume changing materials. Steps toward this goal have been made in recent papers by, e.g., Schmid et al. (2012) and Valentín et al. (2013), where volumetric changes in finite element simulation of simple cubes, as well as a general three dimensional (3-D) framework for G&R of an axisymmetric

* Corresponding author. Tel.: +46 70 260 7885.
E-mail address: thomas@tseeriksson.se (T.S.E. Eriksson).

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cylinder, have been shown. Here, we use an existing model for arterial G&R as a basis (Watton et al., 2004), extending it to a thick-walled, 3-D and volume changing model and discuss the implications of fundamental assumptions about the growth process.

First, we briefly outline the structure, biology and mechanical model of the arterial wall. Arteries consist of three layers, the intima, the media and the adventitia, going from inner to outer layer. For a healthy artery, the main structural constituents are elastin fibres and vascular smooth muscle cells (VSMCs), mostly found in the media, and collagen fibres found in the media and the adventitia. The components are embedded in a ground substance, which is a hydrophilic gel rich in proteoglycans, Wagenseil and Mecham (2009). Following the approach by Holzapfel et al. (2000) on arterial modelling, we neglect the influence of the intimal layer, which under young and healthy conditions bears almost no load, as well as the active mechanical contribution from the VSMCs. Hence, we only model the passive vascular response.

During the progression of many vascular diseases, such as cerebral aneurysm or abdominal aortic aneurysm, the dilation of an artery is associated with a significant loss of the elastin constituent (He and Roach, 1994). Simultaneously, the collagen fabric adapts (via G&R) to compensate for the loss of load borne by the elastinous constituents and the changes to the geometry. Collagen growth relates to changes in the total mass of collagen whereas collagen remodelling relates to changes in the natural reference configurations that the fibres are recruited to load bearing. In general, it is assumed that collagen fibres, which are in a continual state of deposition and degradation, are configured to the artery in the loaded configuration in a small state of stretch. Consequently, as the geometry of the artery changes the reference configurations of the fibres evolve. As presented in Section 2, we follow the G&R approach proposed by Watton et al. (2004), but follow Schmid et al. (2010, 2013) and extend the formulation to a thick-walled model of the arterial wall which incorporate non-volume changing (isochoric) measures as well as an invariant basis. Moreover, we propose and implement a novel approach for modelling volumetric growth and remodelling (VGR) for a mixture of constituents.

A recent study by Valentín et al. (2013) showed as a special case for verification that a pressurised cylindrical model of an artery (consisting of elastin only) will decrease in radius if elastin mass degradation is simulated by enforcing (isotropic) volumetric loss. This is, perhaps, counter intuitive of what one would expect to happen to a mature artery which is subject to elastin degradation or fragmentation. Consequently, the numerical approach to simulate elastin degradation when using a computational framework which can simulate volumetric adaption may need careful attention. In this study, we propose a novel formulation to model the volumetric changes that occur to the arterial wall as a consequence of local changes in mass of constituents. Mass changes can be implemented by changes in density (with fixed volume) or changes in volume (with fixed density) or a combination of the two. We observe that whether the artery shrinks or enlarges depends on which assumption for constituent growth is used, i.e. constant individual density (CID) or adaptive individual density (AID); CID means that the density of a given constituent in a material does not change when its mass is increased or decreased; AID means that the volume of a constituent remains constant when its mass is altered. We will detail these approaches in Section 2, where we define the model components used for VGR of a fibre composite, here specialised for an artery. In Section 3 we show numerical examples illustrating the main features of the model and how it can be applied on AAA evolution. We discuss the proposed model and the underlying biological assumptions in Section 4 followed by some concluding remarks in Section 5.

2. Material and methods

The model of the arterial wall simulates the instantaneous mechanical response due to applied loading and the long-term mechanical response due to G&R of constituents. Consequently, it is convenient to employ two separate time scales: one short time scale, t in seconds, for mechanical equilibrium where the material is assumed to be incompressible and one long time scale, τ in years, for G&R. The kinematics needed and used in this paper are outlined in Section 2.1 and a strain-energy function used for describing the individual mechanical responses of the constituents relative to their natural reference configuration of an artery is shown in Section 2.2. A volumetric function that is used in a penalty scheme to enforce the volume changes is shown in Section 2.3 followed by the material and spatial stress tensors in Section 2.4. Section 2.5 is then devoted to the development of the VGR formulations including mass, density and volumetric changes.

2.1. Kinematics

Using the deformation gradient $\mathbf{F}$, the right and left Cauchy–Green tensors are $\mathbf{C}=\mathbf{F}^{\mathrm{T}}\mathbf{F}$ and $\mathbf{b}=\mathbf{F}\mathbf{F}^{\mathrm{T}}$, respectively. In the short time scale, which governs mechanical equilibrium, the volume ratio (or Jacobian determinant) between $\mathrm{d}V(t_0)$, where t_0 is the time in the reference configuration, and $\mathrm{d}V(t)$, where t is the time in the current configuration, is given by $J(t)=\mathrm{d}V(t)/\mathrm{d}V(t_0)$ or equivalently $J(t)=\det\mathbf{F}>0$. An isochoric deformation gradient, $\overline{\mathbf{F}}$, originating from Flory (1961), is obtained from defining

$$\mathbf{F}\equiv(J^{1/3}\mathbf{I})\overline{\mathbf{F}} \quad \text{where } \overline{\mathbf{F}}=J^{-1/3}\mathbf{F} \text{ leading to } \det\overline{\mathbf{F}}=1. \tag{1}$$

The modified (isochoric) right and left Cauchy–Green tensors are now $\overline{\mathbf{C}}=\overline{\mathbf{F}}^{\mathrm{T}}\overline{\mathbf{F}}=J^{-2/3}\mathbf{C}$ and $\overline{\mathbf{b}}=\overline{\mathbf{F}}\overline{\mathbf{F}}^{\mathrm{T}}=J^{-2/3}\mathbf{b}$, respectively. Isochoric material invariants are further constructed as $\overline{I}_1=\overline{\mathbf{C}}:\mathbf{I}$, $\overline{I}_2=1/2(\overline{I}_1^2-\overline{\mathbf{C}}^2:\mathbf{I})$ and $\overline{I}_3=\det(\overline{\mathbf{C}})=1$, where $\mathbf{I}$ is the second order identity tensor. For an anisotropic material, with two fibre families ($i=1,2$) in the Lagrangian directions $\mathbf{a}_{0,i}$, we may