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A hierarchical computational model for stretchable interconnects with fractal-inspired designs

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ABSTRACT

Stretchable electronics that require functional components with high areal coverages, antennas with small sizes and/or electrodes with invisibility under magnetic resonance imaging can benefit from the use of electrical wiring constructs that adopt fractal inspired layouts. Due to the complex and diverse microstructures inherent in high order interconnects/electrodes/antennas with such designs, traditional non-linear postbuckling analyses based on conventional finite element analyses (FEA) can be cumbersome and time-consuming. Here, we introduce a hierarchical computational model (HCM) based on the mechanism of ordered unraveling for postbuckling analysis of fractal inspired interconnects, in designs previously referred to as 'self-similar', under stretching. The model reduces the computational efforts of traditional approaches by many orders of magnitude, but with accurate predictions, as validated by experiments and FEA. As the fractal order increases from 1 to 4, the elastic stretchability can be enhanced by ~200 times, clearly illustrating the advantage of simple concepts in fractal design. These results, and the model in general, can be exploited in the development of optimal designs in wide ranging classes of stretchable electronics systems.

1. Introduction

Recent advances in mechanics and materials for stretchable/flexible electronics (Lacour et al., 2005; Khang et al., 2006; Lacour et al., 2006; Jiang et al., 2007, 2008; Sekitani et al., 2009; Rogers et al., 2010; Huang et al., 2012; Yang and Lu, 2013; Duan et al., 2014) and optoelectronics (Kim et al., 2010; Lee et al., 2011a; Lipomi et al., 2011; Nelson et al., 2011) demonstrate that systems with high-performance semiconductor functionality can be realized in forms that allow extreme mechanical deformations, e.g., stretching like a rubber band, twisting like a rope, and bending like a sheet of paper. This class of technology creates many application opportunities that cannot be addressed with established technologies, ranging from "epidermal" health/wellness monitors (Kim et al., 2011b; Kaltenbrunner et al., 2013; Schwartz et al., 2013), to soft surgical

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instruments (Cotton et al., 2009; Yu et al., 2009; Viventi et al., 2010; Graudejus et al., 2012; Kim et al., 2012b), to eyeball-like digital cameras (Ko et al., 2008; Song et al., 2013), to sensitive robotic skins (Someya et al., 2004; Wagner et al., 2004; Mannsfeld et al., 2010; Lu et al., 2012). Many of these stretchable systems exploit a strategy, sometimes known as the island-bridge design (Kim et al., 2008, 2009, 2011b; Ko et al., 2008; Lee et al., 2011b), in which the active devices reside on non-deformable platforms (i.e. islands) with deformable interconnects (i.e. bridges) in between. These bridges provide stretchability, while the islands undergo negligible deformation (usually $<1\%$ strain) to ensure mechanical integrity of the active devices (Kim et al., 2008; Song et al., 2009). The stretchability of a system with a certain filling ratio of islands can be written by

$$\text{stretchability of the system} = (1-\sqrt{\text{filling ratio}})*(\text{stretchability of the interconnect}). \tag{1}$$

where the filling ratio denotes the ratio of area covered by the islands to the entire area of an island-bridge structure. Various types of interconnect technologies have been developed, typically involving planar serpentines (Jones et al., 2004;

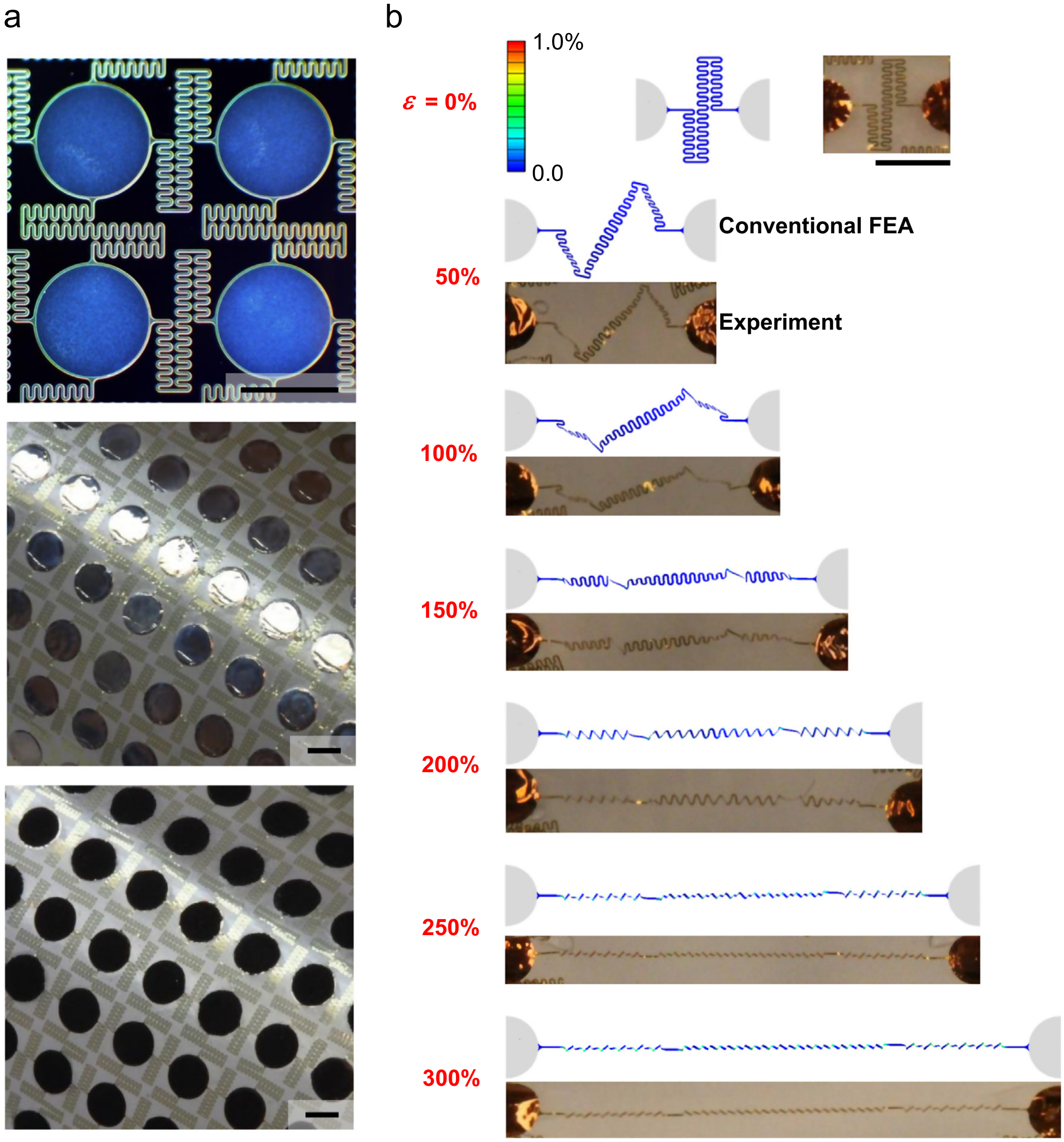

Fig. 1. (a) Optical images of electrode pads and fractal inspired interconnects on a silicon wafer (top panel; top down view; ~4 unit cells), after transfer printing on a sheet of silicone (middle panel; oblique view, in a bent geometry), and with molded slurries of $LiCoO_2$ (bottom panel; oblique view, in a bent geometry), for a stretchable Li-ion battery; (b) optical images and corresponding conventional FEA results of symmetric deformation modes, for various levels of applied tensile strain ε. The scale bars in (a) and (b) are 2 mm. (a and b) Are reprinted with permission from Xu et al. (2013), Copyright 2013, Nature Publishing Group.