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A variational model for fracture and debonding of thin films under in-plane loadings

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ABSTRACT

We study fracture and debonding of a thin stiff film bonded to a rigid substrate through a thin compliant layer, introducing a two-dimensional variational fracture model in brittle elasticity. Fractures are naturally distinguished between transverse cracks in the film (curves in 2D) and debonded surfaces (2D planar regions). In order to study the mechanical response of such systems under increasing loads, we formulate a dimension-reduced, rate-independent, irreversible evolution law accounting for both transverse fracture and debonding. We propose a numerical implementation based on a regularized formulation of the fracture problem via a gradient damage functional, and provide an illustration of its capabilities exploring complex crack patterns, showing a qualitative comparison with geometrically involved real life examples. Moreover, we justify the underlying dimension-reduced model in the setting of scalar-valued displacement fields by a rigorous asymptotic analysis using Γ-convergence, starting from the three-dimensional variational fracture (free-discontinuity) problem under precise scaling hypotheses on material and geometric parameters.

1. Introduction

1.1. Background

Cracking of thin films systems is often experienced in everyday life. Ceramic painted artifacts, coated materials, stickers, paintings and muds are some of the physical systems that exhibit the appearance of complex networks of cracks channeling through the topmost layer. In addition, the phenomenology is enriched by the possible interplay with mechanisms of spontaneous interfacial debonding. Within the three-dimensional multilayer system, although cracks may appear anywhere and with arbitrary geometry, it is a common observation that cracks are either transverse and channeling through the film or planar debonding surfaces at the interface. A comprehensive review of common fracture patterns may by found in Hutchinson and Suo (1992).

Within the framework of classical fracture mechanics, the propagation of crack tip(s) along a pre-defined crack path is obtained through a criterion of critical energy release rate. In their seminal paper, Hutchinson and Suo (1992) provide closed

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form computations of the energy release rate associated to isolated straight or kinked cracks for general layered materials. The concept of steady-state cracking is first formulated as the condition for which the “crack driving force” reaches a value independent of the size of the initial crack, this being the case for cracks that are long compared to the film thickness. Xia and Hutchinson (2000) propose a reduced two-dimensional model for a thin film system as an elastic membrane on an elastic foundation. Then, they investigate the steady-state propagation of isolated cracks and arrays of cracks, illustrate the interaction between parallel or perpendicular neighboring cracks and show, under additional hypotheses, the existence of a particular solution of a crack evolving along an Archimedean spiral. A comparison between the reduced model and the full three-dimensional non-homogeneous layer stack is carried out in Yin et al. (2008), validating the reduced model in the regime of stiff films over a compliant substrate. The presence of an elasto-plastic interface is investigated by McGuigan et al. (2003) and a family of visco-elasto-plastic effective laws for the bonding layer have been analyzed by Handge (2002).

From a numerical standpoint, fracture of thin films has been investigated via phenomenological spring-network models by Crosby and Bradley (1997), Leung and Néda (2000), Sadhukhan et al. (2011), whilst Liang (2003) and Fan et al. (2011) proposed to tackle the problem by means of an extended finite elements discretization. However, XFEM approaches still have difficulties in correctly describing crack branching, coalescence and nucleation. Neither of these works accounts for the interplay between channel cracking and debonding.

In the applied mathematics community, static fractures in single-layer thin films have been investigated by means of a $\Gamma-$convergence analysis that allows the identification of an effective reduced 2D model (Braides and Fonseca, 2001; Bouchitte et al., 2002). Babadjian (2006) studied the quasi-static evolution of cracks in thin films proving the convergence of the full three-dimensional evolution to the reduced two-dimensional one. These results are obtained considering a single-layer system resulting in cracks that are invariant in the thin direction. The dimension reduction of a bilayer thin film allowing for debonding at the interface has been investigated by Bhattacharya et al. (2002), debonding being penalized by a phenomenological interfacial energy paying for the jump of the deformation at the interface. The limiting models are discussed according to the weight of interfacial energy. Rigorous derivations of decohesion-type energies have been given in Ansini et al. (2007) and Ansini (2004) by means of a homogenization procedure. In these works the interfacial energy appears as the limit of a Neumann sieve, debonding being regarded as the effect of the interaction of two thin films through a suitably periodically distributed contact zone. More recently, Dal Maso and Iurlano (2013), Iurlano (2012), Focardi and Iurlano (2014), and Conti et al. (2014) have also derived similar cohesive fracture models by means of an Ambrosio–Tortorelli approximation (Ambrosio and Tortorelli, 1992) involving an internal damage variable. Finally, several works have focused on the quasi-static evolution of debonding problems with a prescribed debonding zone. In particular, Roubíček et al. (2009) modeled the debonding phenomenon through an internal variable representing the volume fraction of adhesive contact between the layers. However, none of these works is able to rigorously justify the models used by the engineering fracture mechanics community to model the cracks of thin film/substrate systems (Hutchinson and Suo, 1992).

1.2. Objectives and organization of the paper

In this paper we investigate the fracture in thin film systems within the framework of variational fracture mechanics (Francfort and Marigo, 1998; Bourdin et al., 2008). Our aim is three-fold: (i) to formulate a two-dimensional variational model for a thin film bonded on a rigid substrate including possible transverse fracture in the film and film/substrate debonding; (ii) to develop a regularized model and show its numerical solutions featuring complex crack patterns with possible coupling between transverse fracture and debonding; and (iii) to justify the emergence of the two-dimensional model as asymptotic limit of a three-dimensional thin film system under precise scaling hypotheses on the geometric and material parameters. Our model considers a geometrically linearized theory and it is applicable only in the case of loads inducing tensile strains. Thin film failure in compression is deeply influenced by several phenomena not included in our analysis, such a buckling and unilateral contact of the crack lips (Audoly and Boudaoud, 2008; Faou et al., 2012).

More generally, our work may be regarded as an attempt to bridge the gap between the mathematically oriented literature and the engineering applications. We apply variational methods to justify the asymptotic behavior of the brittle structure and provide effective techniques for its numerical modeling.

The present paper is composed of two main parts:

(a) *The statement of the mechanical problem and the underlying assumptions* (*Section* 2) *along with the analysis and formulation of the 2D fracture mechanics problem* (*Section* 3). We present the three-dimensional fracture mechanics problem (Section 2) of a thin film bonded to a stiff substrate through a compliant bonding layer. We expose the scaling hypotheses on geometric and material parameters and resume the fundamental properties of the two-dimensional limit model unveiled by the asymptotic analysis. We state in Section 3 the two-dimensional variational problem of fracture of thin films adopting a reduced-dimension model. We shall consider the following limit two-dimensional energy functional:

$$E(u,\Gamma,\Delta)=\underbrace{\frac{1}{2}\int_{\omega\setminus\Gamma}A(\epsilon(u)-\epsilon_0)\cdot(\epsilon(u)-\epsilon_0)\,dx+\frac{1}{2}\int_{\omega\setminus\Delta}\kappa|u|^2\,dx}_{\text{elastic energy}}+\underbrace{g\ \text{length}\ (\Gamma)+G\ \text{area}\ (\Delta)}_{\text{fracture energy}}. \tag{1}$$