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Fully coupled peridynamic thermomechanics

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ABSTRACT

This study concerns the derivation of the coupled peridynamic (PD) thermomechanics equations based on thermodynamic considerations. The generalized peridynamic model for fully coupled thermomechanics is derived using the conservation of energy and the free-energy function. Subsequently, the bond-based coupled PD thermomechanics equations are obtained by reducing the generalized formulation. These equations are also cast into their nondimensional forms. After describing the numerical solution scheme, solutions to certain coupled thermomechanical problems with known previous solutions are presented.

1. Introduction

Thermomechanics concerns the influence of the thermal state of a solid body on the deformation and the influence of the deformation on the thermal state. In many cases, the effect of the deformation field on the thermal state may be ignored. This leads to a decoupled or uncoupled thermomechanical analysis, for which only the effect of the temperature field on the deformation is present. However, the uncoupled thermomechanics may not be satisfactory for certain transient problems. Experimental verification of the influence of the deformation on the thermal state exists (Thomson, 1853; Stanley, 2008). It was shown that an adiabatic solid experiences a temperature drop when it is strained in tension (Chadwick, 1960; Fung, 1965). Also, elastic bodies under tensile loading experience cooling below the yield stress; however, beyond the yield stress the bodies heat up due to the irreversible nature of plasticity (Nowinski, 1978).

Also, the temperature field induced by structural loading may not be uniform. For example, when a beam with an initially uniform temperature is under bending, part of the beam is in tension while the other part is in compression. Due to the thermomechanical coupling, the part of the beam that is in tension cools and the region that is in compression heats up, establishing a thermal gradient. This leads to the onset of heat diffusion. The heat flow is irreversible; thus, some of the mechanical energy supplied to bend the beam is dissipated through its conversion to heat energy. This phenomenon is called thermoelastic damping and it plays a critical role in vibrations and wave propagation.

It is well known that during fracture in metals a plastic region, in which the material has locally yielded, occurs ahead of the crack tip. As a result, the mechanical energy is dissipated as heat and the temperature rises in the local region ahead of the crack tip. A slightly different phenomenon is observed for fracture in polymers. During fracture in polymers, it was experimentally observed that thermoelastic cooling is followed by a temperature rise due to the plastic zone and/or fracture process itself which exposes new surfaces (Rittel, 1998). Consequently, in order to accurately model fracture, especially the crack tip, thermal consideration needs to be taken into account and a coupled thermomechanical analysis becomes

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necessary. The thermal and structural interaction becomes especially important for high-speed impact and penetration fracture problems (Brünig et al., 2011).

The derivation of the classical thermomechanics equation from a thermodynamic perspective did not occur till the mid 1950s (Biot, 1956). Biot used generalized irreversible thermodynamics to formulate the classical thermomechanical laws in variational form, with the corresponding Euler equations representing the coupled momentum and energy equations.

The fully coupled thermomechanical equations based on the classical theory are well established. The classical equations of thermoelasticity are comprised of the deformation equation of motion with a thermoelastic constitutive law and the heat transfer equation with a structural (or deformational) heating and cooling term contributing to the thermal energy. For isotropic materials, the thermoelastic constitutive law includes the thermal stresses, which are related to the temperature gradient, while the structural heating and cooling are dependent on the thermal modulus and rate of dilatation. Depending on the structural idealization, the thermal modulus is defined as

$$\beta_{cl} = (3\lambda + 2\mu)\alpha = \frac{E\alpha}{1-2\nu} \text{ for three dimensions} \tag{1a}$$

$$\beta_{cl} = (2\lambda + 2\mu)\alpha = \frac{E\alpha}{(1-\nu)} \text{ for two dimensions,} \tag{1b}$$

$$\beta_{cl} = (2\mu)\alpha = E\alpha \text{ for one dimension,} \tag{1c}$$

in which E is the elastic modulus, α is the coefficient of thermal expansion, and ν is the Poisson's ratio. The parameters λ and μ are Lamé's constants.

Typically, the strength of coupling is measured via the nondimensional quantity known as the coupling coefficient and defined as

$$\epsilon = \frac{{\beta_{cl}}^2 \Theta_0}{\rho c_v (\lambda + 2\mu)}, \tag{2}$$

for which ρ is the mass density, c_v is the specific heat capacity, and Θ_0 is the reference temperature at which the stress in the body is zero (Nowinski, 1978). The presence of coupling makes the computational solution significantly complicated. If the coupling coefficient, Eq. (2) is small compared to unity, the presence of coupling may be disregarded. The coupling coefficients for metals are significantly lower than those of plastics. Steel, for example, has a coupling coefficient of about 0.011 while certain plastics have a value of $\epsilon = 0.43$.

2. Local theory

Various researchers analytically examined plane waves in thermoelastic solids (Chadwick and Sneddon, 1958; Deresiewicz, 1957). In a one-dimensional formulation, they showed that the presence of thermal and elastic waves, which are dispersed and attenuated. They also studied the effect of frequency on the phase velocity, attenuation, and damping. Later, Chadwick (1962) extended the analysis to two dimensions and investigated the propagation of thermoelastic waves in thin plates. Paria (1958) determined the temperature and stress distribution of a two-dimensional half-space problem using Laplace and Hankel transforms. Laplace transforms have also been used by Boley and Hetnarski (1968) to characterize propagating discontinuities in various one-dimensional coupled thermoelastic problems. Fourier transforms were employed by Boley and Tolins (1962) to determine the mechanical and thermal response of a one-dimensional semi-infinite bar with transient boundary conditions. The major challenge with transform methods is in finding the analytical inverse transforms—in many cases this is not possible and numerical inversion is necessary. Other analytical solution methods have been adopted to solve coupled thermoelastic problems. Soler and Brull (1965) used perturbation techniques and more recently Lychev et al. (2010) determined a closed-form solution by an expansion of the eigenfunctions generated by the coupled equations of motion and heat conduction.

Numerical approximations to the classical thermoelastic equations have been very commonly found using the finite element (FE) method. A transient thermoelastic FE model was developed by Nickell and Sackman (1968) and Ting and Chen (1982). The approximations from their FE model were compared against analytical solutions for various one-dimensional semi-infinite problems. Oden (1969) and Givoli and Rand (1995) developed dynamic thermoelastic FE models. Additionally, Chen and Weng (1988, 1989a, 1989b) modeled various thermoelastic problems such as the transient response of an axisymmetric infinite cylinder and an infinitely long plate using a finite element formulation in the Laplace transform domain. Hosseini-Tehrani and Eslami (2000) presented solutions for thermal and mechanical shocks in a finite domain based on the boundary element method (BEM) in conjunction with the Laplace-transform method in a time domain. They provided results for small time durations (early stages of the shock loads) using the numerical inversion of the Laplace-transform method.

Numerical solution schemes for thermomechanical problems are divided into two categories—monolithic schemes and staggered schemes. In monolithic schemes, the differential equations for different variables are solved simultaneously. On the other hand, for staggered or partitioned schemes, the solutions of the different variables are determined separately. In general, the staggered schemes have been favored over monolithic schemes, as the monolithic systems can be very large, making it unfeasible to solve practical problems. In addition, the mechanical and thermal parts of a thermomechanical