Journal of the Mechanics and Physics of Solids 64 (2014) 44–60

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Non linear constitutive models for lattice materials

Andrea Vigliotti[a], Vikram S. Deshpande[b], Damiano Pasini[a,*]

[a] Department of Mechanical Engineering, McGill University, 845 Sherbrooke St. W, Montreal H3A 2T5, Canada
[b] Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, UK

ARTICLE INFO

Article history:
Received 19 December 2012
Received in revised form
15 October 2013
Accepted 27 October 2013
Available online 8 November 2013

Keywords:
Lattice materials
Periodic cellular materials
Multiscale mechanics
Non linear mechanics

ABSTRACT

We use a computational homogenisation approach to derive a non linear constitutive model for lattice materials. A representative volume element (RVE) of the lattice is modelled by means of discrete structural elements, and macroscopic stress–strain relationships are numerically evaluated after applying appropriate periodic boundary conditions to the RVE. The influence of the choice of the RVE on the predictions of the model is discussed. The model has been used for the analysis of the hexagonal and the triangulated lattices subjected to large strains. The fidelity of the model has been demonstrated by analysing a plate with a central hole under prescribed in plane compressive and tensile loads, and then comparing the results from the discrete and the homogenised models.

1. Introduction

Lattice materials are a class of cellular materials characterised by a regular, periodic microstructure that can be idealised as a network of slender beams or rods. As all cellular materials, they combine properties such as lightness, stiffness, strength and high energy absorbing capabilities that cannot be achieved by uniform fully solid materials (Ashby, 2005; Fleck et al., 2010; Gibson and Ashby, 1999; Banhart, 2001). In addition, due to their regular and controlled microstructure, lattice materials can be designed to fulfil specific requirements, such as prescribed stiffness and strength along given directions and predetermined collapse modes.

Recent advances in manufacturing techniques allow the production of lattice materials from a variety of solid materials, at a very fine scale, with high accuracy and within acceptable costs (Bidanda and Bartolo, 2008; Ramirez et al., 2011; Schaedler et al., 2011). Such technologies make lattice materials a viable option for use in the design of consumer products, and have driven the interest in modelling tools for the analysis of complex components made of lattice materials. Applications that exploit the design of bending dominated lattices for morphing structures, and that focus on stretching dominated lattices suitable for reconfigurable and smart actuated structures (Fleck et al., 2010; Wang et al., 2007; Spadoni and Ruzzene, 2007) typically require modelling lattice materials into the non linear regime. However, the literature on the modelling of the mechanical properties of lattice materials is generally restricted to the geometrically linear regime.

In this paper, we are concerned with the derivation of a constitutive model for the analysis of the geometrically non linear behaviour of lattices. A number of studies have analysed simple topologies and obtained closed-form expressions of the lattice stiffness and strength by solving the equilibrium problem of the unit cell (Gibson and Ashby, 1982; Wang and McDowell, 2004; Gibson et al., 1982; Zhu et al., 1997; Hutchinson and Fleck, 2006). This approach cannot in general be

* Corresponding author. Tel.: +1 514 398 6895; fax: +1 514 398 7365.
E-mail address: damiano.pasini@mcgill.ca (D. Pasini).

0022-5096/$ - see front matter
http://dx.doi.org/10.1016/j.jmps.2013.10.015

extended to include geometrical non linearities, because closed-form solutions are typically unavailable for a beam under large displacement. In other homogenisation approaches (Kumar and McDowell, 2004; Langley, 1996; Suiker et al., 2001; Gonella and Ruzzene, 2008; Elsayed and Pasini, 2010a,b; Vigliotti and Pasini, 2012a,b), the equivalent stiffness of the lattice is determined by comparing selected physical quantities of the discrete lattice, such as the dispersion relation of harmonic waves, or the coefficients of the elastic equilibrium equation, to those of an equivalent continuous medium. These models are necessarily restricted to the linear regime and cannot be extended to consider the effects of geometric or material non linearity. Despite the lack of a specific literature on non linear models for lattices, several works, mainly focused on the modelling of composites and of heterogeneous media, are available. These studies can offer insight into the general framework and theoretical basis for the development of a non linear constitutive model for lattices. Extensive reviews of these works have been produced by Pindera et al. (2009), and by Charalambakis (2010).

The approach presented in this study belongs to the class of representative volume element (RVE) methods (Kouznetsova et al., 2001, 2002, 2004; Van Der Sluis et al., 2000; Terada et al., 2000; Smit et al., 1998; Matsui et al., 2004; Fish and Wagiman, 1993; Mohr, 2005; Ohno et al., 2002), which evaluate the constitutive relationships of a heterogeneous medium from the analysis of a small portion of it. The RVE consists in a limited region of the domain that contains the main microstructural features of the material and responds as the infinite medium, if uniform strain, or stress, and boundary conditions are imposed. All these methods are based on the self-consistent scheme developed by Eshelby (1957), who studied the mechanics of an ellipsoidal inclusion in an infinite matrix with homogeneous boundary conditions. In general, RVE methods resort to a two-scale approach. On one hand, there is the macroscopic finite element model of the component, whose boundary conditions are defined by the general problem, where the material is treated as a homogeneous continuum. On the other hand, there exists the microscopic model of the RVE, which numerically evaluates the stress–strain relationship, whose boundary conditions are generated by the macroscopic model. The RVE model is interrogated at every integration point of the component model, a process that allows the assembly of the macroscopic internal force vector and of the tangent stiffness matrix, as it is done for a fully solid material. These methods, developed for random media, where the RVE is modelled by means of continuous elements, evaluate the macroscopic stress as the average of the microscopic stress on the RVE. Such techniques while feasible for lattice materials (Kouznetsova et al., 2001, 2002, 2004; Van Der Sluis et al., 2000; Ohno et al., 2002) are numerically very cumbersome. It is more natural for these materials to consider RVEs with discrete structural elements, such as beams or shells. However, we typically do not want to carry higher order stresses, such as moments, from the micro- to the macro-scales. For instance, we could not evaluate the contribution to the macroscopic stress of the lattice of a single beam in pure bending, because the stress average over the cross section would be null everywhere along the beam, even though a lattice comprising such elements will sustain a finite macroscopic stress. Other works (Kumar and McDowell, 2004) derive macroscopic constitutive relations for trusses assuming a homogeneous displacement model for the RVE. Such an approach is valid for RVEs that hold central symmetry; nevertheless, if the RVE loses its symmetry during loading, as a consequence of large macroscopic strain, or post-bifurcation, the displacement field of the RVE cannot be described as homogeneous and it should be determined enforcing the equilibrium.

In this study, we use an alternative approach, which allows us to determine the macroscopic stress as the gradient of the strain energy density with respect to the components of the macroscopic displacement gradient. This formulation leads to a compact matrix expression for the macroscopic stress as a function of the macroscopic displacement gradient that can handle both geometrical and material non linearities. The kinematic assumption undertaken here is that the deformation of the lattice periodic directions is congruent with the macroscopic displacement gradient, while the RVE configuration is determined by imposing periodic equilibrium conditions on the RVE. As with any RVE approach, the analysis presented here also assumes periodic deformations and thus cannot account for random spatially distributed imperfections within the lattice material.

The choice of the RVE plays an important role in the framework of a computational homogenisation approach (Gusev, 1997; Kanit et al., 2003; Gitman et al., 2007; Terada et al., 2000; Graham and Yang, 2003; Stroeven et al., 2004). Most of these studies focus on materials with a stochastic microstructure, and are aimed at determining the conditions that ensure a statistical representativeness of the RVE, both for the purpose of numerical homogenisation and for the definition of the size, shape, and number of samples required for experimentally measuring the material properties. Since our focus is on periodic lattices, a natural choice for the RVE is the unit cell, which we intend as the minimal entity capable of generating the lattice. Such choice, however, is not unique; any collection of contiguous unit cells can be used as RVE. Hence, the following question arises: how does the size of the RVE affect the response of the material? For example, Braides (1985) and Müller (1987) have shown that one should not generally expect a finite RVE in the context of finite deformation and that bifurcations can occur at any length scale. Triantafyllidis and Schraad (1998) and Gong et al. (2005) have taken these works ahead by showing explicitly, using Bloch wave boundary conditions, that some bifurcations occur only as the RVE size goes to infinity. This paper addresses this issue of RVE size choice with particular reference to the effect of geometric non linearity. We show that the size of the RVE has no influence on the model prediction, until a bifurcation point is encountered in the load path. After passing bifurcations, the predicted post-bifurcation behaviour depends on the size of the RVE; hence, preliminary investigations should be carried out for a proper selection of the RVE.

The constitutive material model presented in this paper is validated by comparing the results of a discrete and a continuous model of a rectangular plate with a central hole under in-plane loads. In one case, a direct numerical simulation has been carried out on the discrete lattice, whose elements have been individually modelled as beams. In the other case, the domain was modelled with continuous plane stress elements whose constitutive law was numerically evaluated using the homogenisation approach developed in this study.