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Damage–breakage rheology model and solid-granular transition near brittle instability

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ABSTRACT

We develop a continuum-based theoretical framework that describes brittle instability and localization of deformation into a narrow slip zone as a phase transition between damaged solid and granular material. The formulation is based on irreversible thermodynamics of damage and breakage processes, each associated with a single key state variable, and corresponding energy functions for the damaged solid and granular material. Dynamic instability is associated with a critical level of damage in the solid, leading to loss of convexity of the solid energy function and transition to a granular phase associated with lower energy level. Depending on the confining stress and other conditions, the failure process in the generated granular phase may be associated with mode I and fragmentation or mode II and granular flow. The developed model provides a new approach for analyzing in a unified way various aspects of brittle failure and localization of deformation, with evolving elastic moduli, evolving slip rates and evolving material phases. Numerical simulations indicate that the key parameters governing the evolution from a slow failure process to dynamic slip, and the related transition from damaged solid to granular material, can be constrained by laboratory and seismological observations.

1. Introduction

Rocks, ceramics and similar materials, under relatively low pressure and temperature and without a pre-existing large failure zone, respond to differential stress beyond the elastic limit by generation of distributed brittle cracking (damage) in the bulk. The gradual distributed cracking is manifested macroscopically by changes of average elastic properties and related phenomena such as reduction of wave speeds. This process is observed in numerous laboratory fracturing experiments (e.g., Jaeger and Cook, 1979; Lockner et al., 1992; Stanchits et al., 2006) and it can be modeled effectively by various damage rheology frameworks (e.g. Hamiel et al. 2004, 2009; Bhat et al., 2011). When the level of damage reaches a critical value, the material sustains macroscopic brittle instability involving dynamic rupture and localization of deformation. The volume with intense damage around the localized rupture tip is referred to as the process region or zone (e.g., Broberg, 1999). The motion of the process region leaves behind a zone of localized brittle deformation with granulated material that is referred to as the slip zone (e.g. Ben-Zion and Sammis, 2003, and references therein). Various observational and theoretical results suggest that the discussed brittle failure process is associated with a phase transition from a damaged solid prior to localization to a granular phase of material within the generated slip zone (Ben-Zion, 2008, Section 7).

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Initial quantitative results on aspects of the solid-granular transition and related dynamic phenomena are given by Lyakhovsky et al. (2011) using a continuum viscoelastic damage model and Ben-Zion et al. (2011) based on a statistical physics framework. The goal of this paper is to provide a more complete quantitative description of the transition from a stable damaged solid to unstable material that includes a granulated slip zone. The developments are done within the continuum damage model of Lyakhovsky et al. (1997), (2011), augmented by results from a continuum breakage mechanics (Einav, 2007a,b). The developed continuum damage–breakage model accounts for the density of distributed cracking and other internal flaws in damaged solid rocks with a scalar damage parameter, and addresses the grain size distribution of a fluid- or gas-like granular phase with a breakage parameter. The model provides a framework that can be used to analyze the progression of deformation from a stable distributed brittle failure in a solid with evolving elastic moduli, to localization and dynamic rupture in a zone with one or more vanishing elastic moduli. This generalizes related transitions from slow deformation to dynamic rupture on a frictional interface where the process is limited to a surface (e.g., Rice, 1980, 1993), and in volumes governed by plastic with unchanged elastic moduli rheology (e.g., Rudnicki and Rice, 1975; Perrin and Leblond, 1993).

When the damage reaches at a certain region a critical value leading to dynamic instability, the generated failure zone can be associated with a pseudo-liquid or a pseudo-gas granular phase. Under confinement, a pseudo-liquid granular phase is produced and subsequent failure is associated with shear granular flow. With little or no confinement, a pseudo-gas phase is generated and subsequent failure is associated with expanding fragmentation process. Numerical simulations illustrate several aspects of the developed continuum damage–breakage model. A theoretical treatment of the reversed transition from granulated material to damaged solid will be provided in a follow up work.

2. Theory

2.1. General thermodynamic formulation

To describe a solid-granular transition we combine aspects of two formulations: Continuum Damage Mechanics (CDM) and Continuum Breakage Mechanics (CBM). Both are based on irreversible thermodynamics and each introduces an additional state variable: damage parameter $\alpha=[0{:}1]$ in the CDM and breakage parameter $B=[0{:}1]$ in the CBM. The damage parameter of the CDM (e.g., Kachanov, 1986; Rabotnov, 1988; Krajcinovic, 1996; Lemaitre, 1996; Allix and Hild, 2002) connects the evolution of elastic moduli with changes of crack density through a non-dimensional intensive variable characterizing material volumes large enough to allow smooth description of the distribution of internal flaws (e.g., micro-cracks in laboratory specimen). The breakage parameter of the CBM (Einav, 2007a,b) measures the relative distance of the current grain size distribution of a granular material between the initial and ultimate states. The free energy, F_S, of a damaged solid with distributed cracking has additional terms to those characterizing reversible elastic materials (e.g., Lyakhovsky et al., 1997, 2011). The free energy, F_B, of the CBM is assumed to be a linear function of the breakage parameter (Einav, 2007a,b). Following these ideas we write the total free energy of the Continuum Damage–Breakage Mechanics (CDBM) model as a linear superposition of F_S characterizing the solid phase and F_B of the granular phase:

$$F(T,\ \varepsilon_{ij},\ \alpha,\ B)=(1-B)\cdot F_S(T,\ \varepsilon_{ij},\ \alpha)+B\cdot F_B(T,\ \varepsilon_{ij}). \tag{1}$$

Here T is the temperature and $\varepsilon_{ij}=\varepsilon_{ij}^{(t)}-\varepsilon_{ij}^{(p)}$ is the elastic strain tensor given by the difference between the total and permanent irreversible deformation. As mentioned, α and B are the damage and breakage variables, respectively. The linear superposition of the solid and granular energy terms (1) preserves the linear dependence of the total energy on the breakage parameter, and assures the convexity of the total energy if one or both F_S and F_B are convex and B varies between zero and one. According to the general idea of Maxwell visco-elastic rheology, the energy function of the system depends only on the difference between the total and irreversible strain components rather than having explicit connections to both (e.g., Malvern, 1969). The irreversible strain accumulation may be neglected for the solid phase, but it becomes essential for the post-failure flow of the granular phase. Details of the energy form, kinetics of the evolving damage ($d\alpha/dt$) and breakage (dB/dt) parameters, as well as relations controlling the rate of irreversible strain accumulation ($d\varepsilon_{ij}^{(p)}/dt$), are discussed in the following sections.

2.2. Free energy of the solid phase

For the solid phase we use the energy form previously discussed in Lyakhovsky et al. (1997) and Hamiel et al. (2011), incorporating two Hookean quadratic terms and a third term that couples volumetric and shear strain:

$$F_S(\varepsilon_{ij},\alpha)=\frac{1}{\rho}\left(\frac{\lambda}{2}I_1^2+\mu I_2-\gamma I_1\sqrt{I_2}\right) \tag{2}$$

where λ and μ are Lame constants, $I_1=\varepsilon_{ij}\delta_{ij}$ and $I_2=\varepsilon_{ij}\varepsilon_{ij}$ are the first and second invariants of the strain tensor ε_{ij}, and γ is an additional modulus of a damaged solid.