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Relating pore shape to the non-linear response of periodic elastomeric structures

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ABSTRACT

By introducing a periodic array of pores in an elastic matrix, instabilities with wavelengths that are of the order of the size of the microstructure can be triggered. Interestingly, these instabilities can be utilized to design a novel class of responsive materials. Possible applications include materials with unusual properties such as negative Poisson's ratio, phononic and photonic switches and colorful and reconfigurable displays.

Although shape plays an important role in the design and performance of periodic materials, so far only the non-linear response of structures with circular and elliptical pores has been investigated and the effect of the pore shape on the structural response has not yet been explored. Here, we numerically explore the effect of pore shape on the non-linear response of a square array of pores in an elastomeric matrix. Our results show that pore shape can be used effectively to design material with desired properties and to control attractive features of soft porous systems, such as their stiffness, critical strain and negative Poisson's ratio.

1. Introduction

Materials with significant porosity, generally termed cellular solids, exhibit properties that differ from those of their solid counterparts and have a large number of uses in mechanical and thermal applications (Gibson and Ashby, 1999; Wadley, 2006). In particular, cellular solids are used to design lightweight structures (Queheillalt and Wadley, 2005), to maximize energy absorption (Wierzbicki and Abramowicz, 1983; Papka and Kyriakides, 1994) and for acoustic damping (Verdejo et al., 2009). The connections between the architecture of such materials and their macroscopic properties have been investigated by many researchers (Gibson and Ashby, 1999; Wadley, 2006; Evans et al., 2001; Ashby and Bréchet, 2003). Moreover, the non-linear stress–strain behavior of cellular solids has been of particular interest (Gibson and Ashby, 1999; Papka and Kyriakides, 1998; Triantafyllidis and Schraad, 1998; Ohno et al., 2002; Tantikom et al., 2005; Chung and Waas, 2002). Under compression, elastic foams deform linearly to a strain of about 5%. Then, their cell walls buckle and they collapse at a nearly constant stress until contact between cell walls occurs, giving rise to the final steep portion of the stress–strain curve. Polymeric and metallic foams have similar stress–strain curves, but after instability they deform plastically at nearly constant stress until the cell walls touch.

Although traditionally instabilities have been viewed as an inconvenience, buckling need not to be deleterious: buckling plays an important role in the morphogenesis of some plant parts (Steele, 2000); the surface pattern of a dehydrated fruit is dominated by buckling (Yin et al., 2008) and buckling caused by swelling increases the leave motility in the Venus flytrap (Forterre et al., 2005). Inspired by nature, researchers have recently demonstrated instabilities to be instrumental in controlling adhesion (Chan et al., 2008), facilitating flexible electronics (Rogers et al., 2010), fabricating micro-fluidic structures (Khare et al., 2009), controlling surface wettability (Chung et al., 2007), providing means for micro- and nano-patterning and

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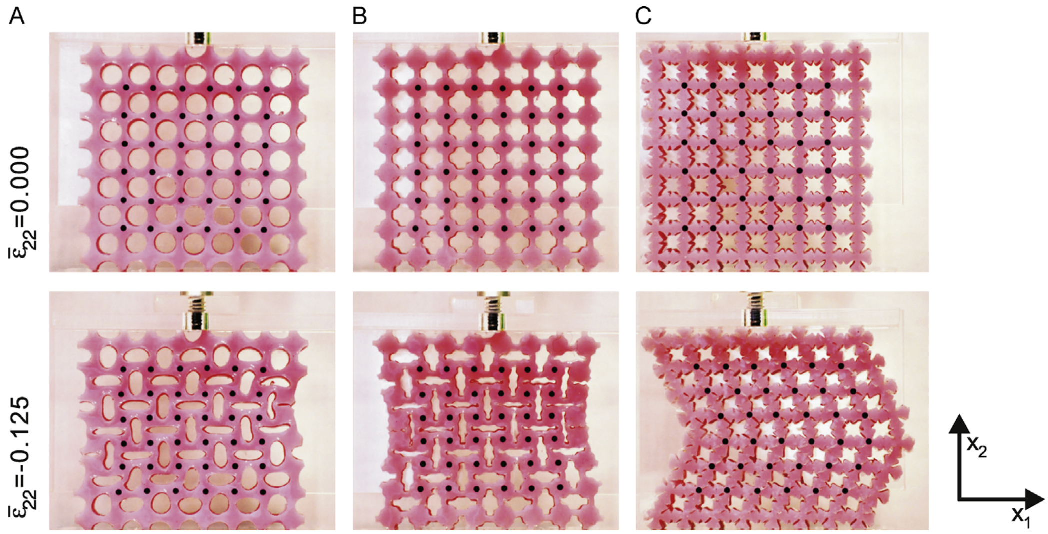

Fig. 1. Experimental images of three periodic structures with differently shaped pores loaded under uniaxial compression (Overvelde et al., 2012). All structures are characterized by 50% porosity and the engineering strain $\varepsilon_{22} = -0.125$ is applied. The shape of the pores is found to strongly affect the instability. In structures A and B the critical instability is characterized by a short wavelength and leads to the formation of a checkerboard pattern. In contrast, a buckling mode with a wavelength equal to the size of the sample is observed in structure C, reminiscent of the twinning observed in austenite to martensite phase transformations in shape memory alloys.

designing optical micro-devices (Yoo et al., 2002; Zhang et al., 2008), designing active micro-hydrogel devices (Lee et al., 2010) and reversible encapsulation systems (Shim et al., 2012).

2D periodic porous structures recently attracted considerable interest because of the dramatic transformations of the original geometry that may be observed as the result of mechanical instabilities. Upon reaching a critical applied deformation, a square array of circular pores in an elastomeric matrix is found to suddenly transform into a periodic pattern of alternating, mutually orthogonal ellipses (Michel et al., 2007; Mullin et al., 2007; Bertoldi et al., 2008; Zhang et al., 2008). Remarkably, it has been demonstrated that these instabilities provide opportunities for fabrication of complex microstructures (Zhang et al., 2008) and for the design of materials with unusual properties such as negative Poisson's ratio (Bertoldi et al., 2010), phononic (Jang et al., 2009) and photonic switches (Krishnan and Johnson, 2009) and reprogrammable colorful displays (Li et al., 2012).

Here, we focus on 2D elastomeric porous structures and investigate the effect of pore shape on their non-linear behavior. While it has been recently shown that the pore arrangement (Triantafyllidis et al., 2006; Michel et al., 2007; Shim et al., 2013), the porosity of the solid (Bertoldi et al., 2010) and the loading conditions (Michel et al., 2007) have a strong effect on the stability of the system, the goal of this work is to provide a deep understanding in the effect of pore shape on the global response of the structure. Since it has been recently demonstrated that pore shape has a strong effect on the instability, affecting not only the point where instability occurs, but also its wavelength (Overvelde et al., 2012) (see Fig. 1), we conduct a systematic numerical study to identify the effect of pore shape on the non-linear response of the periodic elastomeric structures, while keeping the porosity, hole arrangement and loading conditions fixed.

The pore shape is found to provide a convenient parameter to control not only the initial stiffness and the critical strain, but also attractive features of soft porous systems, such as their negative Poisson's ratio.[1] Our results show that the pore shape can be used effectively to design material with desired properties and pave the way for the development of a new class of soft, active and reconfigurable devices over a wide range of length scales.

The paper is organized as follows. First, the family of pore shapes investigated in this study is presented in Section 2. Then, in Section 3 the numerical analyses that are used to investigate the non-linear response of infinite periodic and porous structures are introduced. Finally, numerical results are presented and discussed in Section 4, highlighting the effect of the pore shape on the macroscopic response of the periodic elastomeric structures.

2. Microstructure

In this study, we consider a square array of pores in an elastomeric matrix and focus on pores with four-fold symmetry. Taking the circular pore shape as a starting point, we make use of Fourier series expansion to alter their contour according to

$$x_1 = r(\theta)\cos\theta, \quad x_2 = r(\theta)\sin\theta,$$
$$\text{with } r(\theta) = r_0[1 + c_1\cos(4\theta) + c_2\cos(8\theta)], \tag{1}$$

[1] Although Poisson's ratio is rigorously defined in the framework of linear elasticity, here we extend the concept to finite elasticity and use it to quantify the lateral deformation of the material.