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High strain rate response of rubber membranes

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ABSTRACT

This paper investigates the propagation of axisymmetric waves of finite deformation in polyisoprene rubber membranes subjected to high speed impact. High speed photography is used to monitor the motion and to determine the evolution of stretch and particle velocity in membranes at impact speeds of up to 160 m/s, producing a maximum stretch >8. A constitutive model is developed through a semi-inverse method correlating experimental results with simulations. The potential for formation of wrinkles is also addressed.

1. Introduction

The challenge of modeling the constitutive behavior of rubbers and elastomers is long-standing. At present a number of phenomenological models exist that capture the quasi-static response over certain strain ranges under specific load conditions (for example, the models of Mooney–Rivlin, Ogden, Gent, Arruda–Boyce, etc.). As the strain-rate is increased, however, these models are no longer adequate. Niemczura and Ravi-Chandar (2011a,b,c) building on the pioneering work of von Karman and Duwez (1950), developed a dynamic stretching experiment and found that a simple power-law type model could capture the uniaxial tensile behavior of polyisoprene rubber at strain-rates between 500 and $10^4\,s^{-1}$ for stretches between one and four. In order to develop a robust description of the material behavior however, the response must be extended to larger stretches under a range of strain-rates. Such a model would likely come in the form of a description of the energy function of the material. The goal of this paper is to calibrate such a model under one loading scenario, dynamic stretching for example, and then demonstrate that the model is able to predict the behavior under other loading conditions such as biaxial tension. In order to validate any description of the high strain-rate behavior of rubbers and elastomers, experiments must be performed under loading conditions other than the one used for calibration of the model, and hence the need for the dynamic membrane impact test considered in the present work.

A number of studies have addressed the high strain-rate deformation of a membrane from a number of different angles. Hallquist and Feng (1975) produced some of the earliest analytical work describing the dynamic response of a Mooney–Rivlin membrane subjected to a step pressure load using a Rayleigh Ritz procedure which minimized the potential energy of the deformed configuration of the membrane and obtained the shape variation with time. Early high speed imaging of dynamic membrane deformation along with prediction of transverse wave speed can be found in the work of Farrar (1984). In that study, rubber membranes were impacted with projectiles traveling at 12 m/s. The limitation on impact speed was due to the assumption of small deformations in the wave propagation analysis used to predict the deflection. In this

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restricted case, they were able to match the recorded out of plane deflection reasonably well. Haddow et al. (1992) presented an analytical study of dynamic deformation of membranes under finite strains using the method of characteristics (MC) to model the wave propagation. This work, in which the material was assumed to obey a Mooney–Rivlin constitutive model, did not include any comparison of experimental results with their predictions. In this article, we use high speed imaging to record the large axisymmetric deformation of polyisoprene membranes subjected to normal impact with speeds in the range of 50–150 m/s. These experimental measurements are used to explore a constitutive model by comparing the measured kinematic quantities of deflection, strain, and particle velocity with those predicted by simulations.

The choice of material models must include the possibility of stiffening at high strains. This stiffening is observed in quasi-static tension tests and is attributed to the generation of orientation with stretching, as opposed to simply the unraveling, of the polymer chains and is expected to occur in some form at large stretches even during high strain-rate deformation. The inflection in the stress strain curve which results from this stiffening however, presents the possibility of tensile shock wave formation.

All of the above assumes axisymmetric deformation; however, during several tests wrinkles were observed in the membrane, clearly invalidating the assumed symmetry. The wrinkles initiated at the transverse wave front, and grew radially as the deformation progressed. Vermorel et al. (2009) investigated the formation of these wrinkles during low speed membrane impacts. Through simulation, we explore the possible reason for the appearance of wrinkles only in some of our tests.

This article is organized as follows: the potential constitutive models used to describe the dynamic response of the polyisoprene membrane are described in Section 2. The quasi-linear system of equations that govern axisymmetric wave propagation in a membrane is reviewed in Section 3. The method of characteristics (MC) used to solve the membrane problem is described in Section 4. The experimental setup and the diagnostic tools are described in Section 5. The response of the membrane, determined through high-speed photography, is discussed in Section 6. In Section 7, the measured response of the membrane is compared to the calculated response, and in the process the parameters of the constitutive model are extracted. The observed wrinkling is described, and underlying reasons for its formation are explored through simulations in Section 8. Finally, the key findings are summarized in Section 9.

2. Polyisoprene rubber

The dynamic response of polyisoprene rubber used in this work has been examined by Niemczura and Ravi-Chandar (2011a) who found that at strain rates greater than 500 s^{-1}, the rate dependence of the material saturated and therefore the uniaxial tensile behavior of material could be captured reasonably well with a simple power-law type model with the form $\sigma = \mu\gamma^n$. A comparison between this rate saturated power law and the quasi-static tensile behavior of rubber is shown in Fig. 1 ($\mu = 1$ MPa, $n = 0.5$), a recreation of Fig. 7 from Niemczura and Ravi-Chandar (2011a). In an attempt to generalize this power law to multiaxial loading, it was found that the dynamic uniaxial tensile behavior can also be closely matched by a material with the Mooney–Rivlin strain energy density

$$W = \mu_1(I_1 - 3) + \mu_2(I_2 - 3) \tag{1}$$

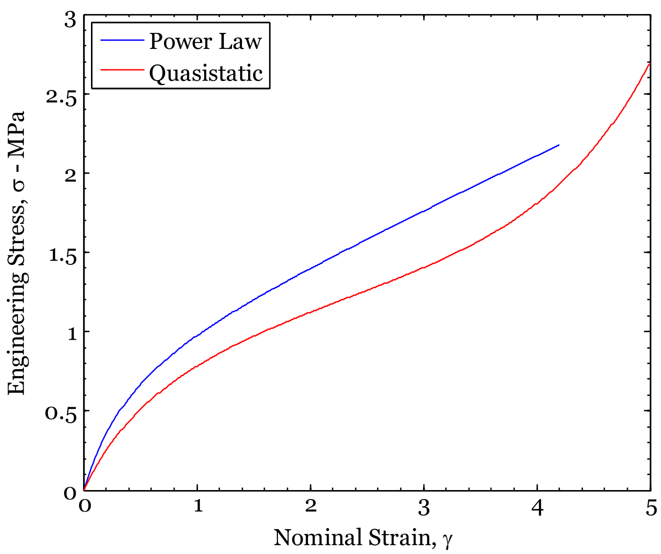

Fig. 1. Comparison of the measured uniaxial tensile response of polyisoprene rubber during quasistatic loading (red) to the power law model (blue) used to capture the rate saturated behavior. Niemczura and Ravi-Chandar (2011a). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)