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A finite strain thermo-viscoelastic constitutive model to describe the self-heating in elastomeric materials during low-cycle fatigue

C. Ovalle Rodas [a,b,c], F. Zaïri [a,b,*], M. Naït-Abdelaziz [a,b]

[a] Univ Lille Nord de France, F-59000 Lille, France
[b] Université Lille 1 Sciences et Technologies, Laboratoire de Mécanique de Lille (LML), UMR CNRS 8107, F-59650 Villeneuve d'Ascq, France
[c] Universidad Rafael Landivar, Campus de Quetzaltenango, 14 Avenida 0-43 zona 3, 09000 Quetzaltenango, Guatemala

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ABSTRACT

A thermo-visco-hyperelastic constitutive model, in accordance with the second thermodynamics principle, is formulated to describe the self-heating evolution in elastomeric materials under cyclic loading. The mechanical part of the model is based upon a Zener rheological representation in which the specific free energy potential is dependent on an added internal variable, allowing the description of the time-dependent mechanical response. The large strain mechanical behavior is described using a Langevin spring, while the continuous stress-softening under cyclic loading is taken into account by means of a network alteration kinetics. The thermo-mechanical coupling is defined by postulating the existence of a dissipation pseudo-potential, function of the viscous dilatation tensor. The proposed model is fully three-dimensional and is implemented into a finite element code. The model parameters are identified using experimental data obtained on a styrene-butadiene rubber under a given strain rate for different strain conditions. Predicted evolutions given by the model for other strain rates are found in good agreement with the experimental data.

1. Introduction

Elastomeric materials are often submitted to cyclic loading conditions in the automotive and aeronautical industries. During cyclic loading and depending on the boundary conditions, the temperature evolves as a consequence of the viscous properties of the material. The self-heating is a phenomenon that depends on the strain rate (Lion, 1997a), on the maximum strain (La Rosa and Risitano, 2000) and, obviously, on the geometry of the test sample (Ayoub et al., 2012).

Since the material behavior depends on the temperature of the sample, the development of a thermo-mechanical constitutive model able to describe the temperature evolution under fatigue loadings is necessary. The model must take into consideration not only the properties associated with the mechanical behavior of the material, namely:

- Non-linear elastic behavior when it is submitted to large strains,
- strain rate dependence,
- relaxation to an equilibrium state for a given strain, and
- stress-softening,

but also the aforementioned parameters (strain rate, maximum strain,…).

* Corresponding author at: Université Lille 1 Sciences et Technologies, Laboratoire de Mécanique de Lille (LML), UMR CNRS 8107, F-59650 Villeneuve d'Ascq, France. Tel.: +33 328767460; fax: +33 328767301.
E-mail address: fahmi.zairi@polytech-lille.fr (F. Zaïri).

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Recently, different approaches describing the thermo-mechanical behavior of different kinds of materials submitted to a cyclic loading have been proposed. The thermo-mechanical coupling description is generally based upon three approaches. The first approach is purely phenomenological. Following this framework, the stored energy quantity of cold work (Kamlah and Haupt, 1998), the fatigue limit of materials related to a sudden self-heating (La Rosa and Risitano, 2000; Fargione et al., 2002; Le Saux et al., 2010), and the temperature evolution as a function of the number of cycles (Doudard and Calloch, 2009) have been described. The second approach is based upon the numerical solution of the problem by means of analytical or finite element (FE) methods. This approach has been used to describe the self-heating and the mechanical parameters evolution with the temperature in polymers submitted to compressive cyclic loading (Molinari, 1996; Rittel, 2000). Hakannson et al. (2005) have proposed a large strain thermo-plastic model, in which the kinematic and isotropic hardenings have been taken into account, leading to satisfactory results when compared to experimental data. Recently, neglecting the thermo-mechanical coupling, the description of the self-heating behavior in polymers under cyclic loading has been achieved using simulation algorithms, implemented into a FE code (de Cazenove et al., 2012; Pichon et al., 2012). The third approach is based on a variational formulation of the thermo-mechanical problem. This approach offers an alternative solution to describe the thermo-visco-plastic behavior of metallic materials (Yang et al., 2006; Stainier and Ortiz, 2010), and the predictions in terms of self-heating rate due to the dissipation were found accurate. Using a variational model similar to that developed by Stainier and Ortiz (2010), Canadija and Mosler (2011) solve the variational problem by means of a completely implicit time integration scheme. In the case of elastomeric materials, Boukamel et al. (2001) have proposed to solve the thermo-mechanical problem by means of a variational formulation applied to the rheological model of Poynting–Thomson. Its solution was numerically obtained after implementation into a FE code (Meo et al., 2002).

Thermo-mechanical constitutive models in accordance with the dissipation principle have been already proposed (Lion, 1997a, 1997b; Boukamel et al., 2001; Hakannson et al., 2005; Yang et al., 2006; Stainier and Ortiz, 2010); however, except (Lion, 1997b) and (Boukamel et al., 2001), these models generally deal with metallic materials, thus restricting their application fields. Even the thermo-mechanical model proposed by Lion was reported in several papers (Lion, 1996, 1997a, 1997b), its application to the self-heating is restricted to a demonstration of the existence of a dissipative self-heating phenomenon due to inelastic deformations. Similarly, the model proposed by Boukamel et al. (2001) is limited to describe the self-heating of an elastomeric piece submitted to shear loading, without validating it for other loading paths.

In this paper, a large strain thermo-viscoelastic constitutive model is developed to describe the self-heating in elastomeric materials during low-cycle fatigue loading. A Zener type rheological model is used to describe the mechanical behavior of elastomeric materials, the strain response being decomposed into two components:

- An equilibrium component that reflects the long time material behavior,
- a viscous component that describes the non-linear time-dependent deviation from the equilibrium response.

The stress-softening phenomenon is taken into account by means of a network alteration kinetics as proposed by Ayoub et al. (2011, in press). In the thermodynamic scheme, the specific free energy potential is defined by introducing, in addition to the usual observable variables ($\boldsymbol{F}$, θ), the viscous dilatation tensor related to the time-dependent deviation from the equilibrium. The existence of a dissipation pseudo-potential, function of the viscous dilatation tensor, is postulated. The associate specific free energy is defined using a Langevin formulation. The proposed model was implemented into a FE code, and the same mechanical and boundary conditions regarding the experimental tests were simulated. Finally, when comparing the predicted results with the experimental data for different strain rate and strain conditions, a good agreement is observed.

The paper is organized as follows: in Section 2, the formulation of the constitutive model including the thermo-mechanical coupling is presented. The experimental methods and the parameters identification procedure are presented in Section 3. In Section 4, the implementation of the model algorithm is described in a general way. The validation of the model regarding the experimental data is presented in Section 5. Finally, the concluding remarks are given in Section 6.

2. Model formulation

The proposed model deals with the non-linear time-dependent behavior of rubber materials. Following a Zener rheological representation, the constitutive model considers the stress–strain response as resulting of two polymer networks acting in parallel. The schematic representation of the model is shown in Fig. 1 in which it is considered that the overall resistance to deformation is the sum of an equilibrium response A (non-linear elastic spring) and a time-dependent deviation with regard to the equilibrium state B (non-linear elastic spring in series with a viscous dashpot).

2.1. Kinematics

The model is based upon continuum mechanics and developed using the finite strain kinematic framework. A key quantity in this kinematic framework is the deformation gradient defined as: $\mathbf{F} \equiv \partial \mathbf{x}/\partial \mathbf{X}$, $\mathbf{x}$ being the position in the current configuration and $\mathbf{X}$ the position in the reference configuration of a given material point. Since a parallel scheme is used, the mathematical representation of the constitutive model is based upon the additive partition of the overall resistance into an