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Geometrically nonlinear higher-gradient elasticity with energetic boundaries

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Abstract

The objective of this contribution is to formulate a geometrically nonlinear theory of higher-gradient elasticity accounting for boundary (surface and curve) energies. Surfaces and curves can significantly influence the overall response of a solid body. Such influences are becoming increasingly important when modeling the response of structures at the nanoscale. The behavior of the boundaries is well described by continuum theories that endow the surface and curve with their own energetic structures. Such theories often allow the boundary energy density to depend only on the superficial boundary deformation gradient. From a physical point of view though, it seems necessary to define the boundary deformation gradient as the evaluation of the deformation gradient at the boundary rather than its projection. This controversial issue is carefully studied and several conclusions are extracted from the rigorous mathematical framework presented.

In this manuscript the internal energy density of the bulk is a function of the deformation gradient and its first and second derivatives. The internal energy density of the surface is, consequently, a function of the deformation gradient at the surface and its first derivative. The internal energy density of a curve is, consequently, a function of the deformation gradient at the curve.

It is shown that in order to have a surface energy depending on the total (surface) deformation gradient, the bulk energy needs to be a function of at least the first derivative of the deformation gradient. Furthermore, in order to have a curve energy depending on the total (curve) deformation gradient, the bulk energy needs to be a function of at least the second derivative of the deformation gradient. Clearly, the theory of elasticity of Gurtin and Murdoch is intrinsically limited since it is associated with the classical (first-order) continuum theory of elasticity in the bulk. In this sense this contribution shall be also understood as a higher-gradient surface elasticity theory.

Keywords: Higher-gradient elasticity, Surface elasticity, Nano-materials, Cosserat continua, Generalized continua

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