Journal of the Mechanics and Physics of Solids 61 (2013) 2680–2691

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Effective conductivities of thin-interphase composites

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ARTICLE INFO

Article history:
Received 18 February 2013
Received in revised form
15 August 2013
Accepted 23 August 2013
Available online 8 September 2013

Keywords:
Multiphase composites
Thin interphases
Effective conductivity

ABSTRACT

A method is presented for approximating the effective conductivity of composite media with thin interphase regions, which is exact to first order in the interphase thickness. The approximations are computationally efficient in the sense the fields need to be computed only in a reference composite in which the interphases have been replaced by perfect interfaces. The results apply whether any two phases of the composite are separated by a single interphase or multiple interphases, whether the conductivities of the composite phases are isotropic or anisotropic, and whether the thickness of an interphase is uniform or varies as a function of position. It is assumed that the conductivities of the interphase materials have intermediate values as opposed to very high or very low conductivities.

1. Introduction

Composite media with thin interphases between adjacent phases are used widely in engineering, biomedical, and technological applications. Examples include coated fiber or coated particle reinforced materials and composites with adhesively bonded joints, see for instance Kim and Mai (1998), Banea and Da Silva (2009) and Ramakrishna et al. (2001). In this work we consider the problem of computing the effective conductivity in a composite with thin interphase regions between some of the composite's constituents, see Fig. 1. The fact that the composite has a thin interphase (or interphases) makes the numerical computations of the fields and the effective properties a difficult task. Therefore, approximate models of thin interphase composites that do not require solving for the fields inside the interphase regions are desirable, (Benveniste, 2012).

There has been an extensive study of interface models, in which an interphase region between two media is replaced by an interface, allowing direct contact of the media, along with appropriate interface conditions. This interface is usually referred to as an imperfect interface when either the potential or the normal flux component has a jump discontinuity across the interface. The effective conductivity in composites with highly conducting or poorly conducting imperfect interfaces has been studied by Benveniste and Miloh (1986), Benveniste (1987), Torquato and Rintoul (1995), Lipton and Vernescu (1996), Lipton (1997), Miloh and Benveniste (1999), Lipton and Talbot (2001) and Le-Quang et al. (2010), among others. Numerical methods for computing the effective conductivity in composites with highly conducting or poorly conducting imperfect interfaces have been recently developed by Yvonnet et al. (2008, 2011).

Although the majority of studies in the literature of thin interphases focus on the case when there is a high contrast between the conductivities of the interphase and its adjacent media, there have been some works that deal with intermediate values of conductivities without the high-contrast assumption. Notable examples of such studies include the

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http://dx.doi.org/10.1016/j.jmps.2013.08.022

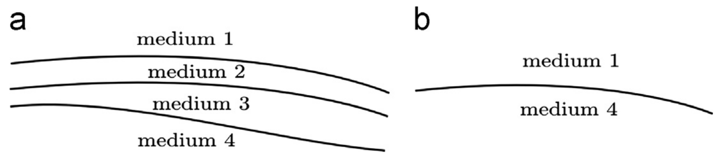

Fig. 1. (a) A composite in which media 2 and 3 form an interphase region between media 1 and 4. (b) A reference composite in which the interphases of the composite in (a) have been replaced by an interface.

work of Hashin (2001) and the works of Benveniste (2006a, 2006b, 2012) and Benveniste and Baum (2007). In the work of Hashin (2001) an interface model is introduced, in which the interphase's conductivity is arbitrary ranging from zero to infinity, and the effect of the interphase on the effective conductivity is discussed. However, this work is restricted to having isotropic and homogeneous phases and the thickness of the interphase is assumed to be constant. In the above studies by Benveniste et al. two models of thin interphase materials are developed of which one is an interface model with the interphase replaced by an interface. In the second model the geometry of the thin interphase is left intact and is characterized by conditions in which the fields are evaluated in the adjacent media at both sides of the interphase and do not involve the fields within the interphase. Some of the interesting features of these models are that the interphase can have anisotropic and inhomogeneous conductivities. However, it is assumed that the thickness of the interphase region is constant. It is also noted that these studies do not discuss the effect of the thin interphase on the effective conductivity. For a more comprehensive review of the topic (on the modeling of thin interphases), we refer the reader to the works given in Benveniste (2006a) and Gu and He (2011).

In this work we present a novel method that provides an approximation to the effective conductivity and combines different desirable features. These include that the method is exact to first order in the interphase thickness and that the method is computationally inexpensive in the sense that it does not involve computing the fields within the interphase region. Additionally, the method applies for homogeneous as well as inhomogeneous interphases, that is whether a single interphase separates two materials or multiple (possibly infinitely many) interphases. Moreover, the conductivities of the composite components, including the interphases, can be isotropic or anisotropic. Furthermore, the geometry of the interphase is arbitrary and its thickness h is not assumed to be constant but can vary slowly, on a length scale large compared to h. However, the total thickness of the interphase region must be sufficiently small in order for our approximation of the effective conductivity to hold. It is also assumed that the interphase conductivities have intermediate values as opposed to very high or very low conductivities. Numerical results for a particular example of a thin-interphase composite show that our approximation agrees very well with the exact effective conductivity when the interphase is thin and has intermediate values of conductivity.

Our method is based on estimating the change in the effective conductivity as the total thickness of the interphases vanishes. Thus we compare the effective conductivity of the composite to a reference composite in which the interphase materials in the original composite have been removed and replaced by a perfect interface, with the usual conditions of continuity of potential and continuity of flux across this interface. Computing the effective conductivity and the fields inside the reference composite is an easier problem that may, for example, be solved using integral equations. We refer to this as the unperturbed problem. The method is based on the following estimate, which holds when the total thickness of the interphases is sufficiently small,

$$\boldsymbol{\sigma}^* \approx \tilde{\boldsymbol{\sigma}}^* = \boldsymbol{\sigma}_0^* + \delta\boldsymbol{\sigma}^*. \tag{1.1}$$

Here $\boldsymbol{\sigma}^*$ is the effective conductivity of the composite with thin interphases, $\tilde{\boldsymbol{\sigma}}^*$ is our approximation, $\boldsymbol{\sigma}_0^*$ is the effective conductivity of the identical composite but with the interphase materials removed and replaced by one material of the two neighboring phases, and $\delta\boldsymbol{\sigma}^*$ is the change in the effective conductivity due to inserting the interphase materials. The formulas that we present and derive in Section 3 show that in order to compute $\delta\boldsymbol{\sigma}^*$ we need to compute the fields only at material interfaces inside the unperturbed problem (or reference composite).

We emphasize that the method introduced here is quite general and can easily be extended to elasticity, piezoelectricity or other coupled field problems. Indeed its basis is the interface-shift formula of Milton (2002) which was developed in this more general context. We have chosen to focus on the conductivity case for simplicity and to make the presentation less abstract.

This paper is organized as follows. In Section 2, a review is given for computing the change in the effective conductivity due to translating an interface. In Section 3, a formula for computing the change in the effective conductivity due to inserting an interphase region is derived. Finally, Section 4 provides numerical results for the effective conductivity of the doubly coated sphere assemblage and shows a comparison between our approximation, the exact result, and two other known approximations.

2. Review

In this section we review how the effective conductivity of a composite changes due to a shift of a phase boundary. The review here is based on the treatment given in Milton (2002, Section 16.6). For completeness of the presentation, we will include a derivation of Milton's interface-shift formula (2.12), in the case of the conductivity of composites.