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# New creep constitutive equation for finite element modelling including transient effects

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## ABSTRACT

Creep experiments are time consuming and expensive; moreover, it is not possible to carry out experiments under the actual service conditions of particular materials due to the very low creep strain rate. However, the process seems to be an ideal field for computer modelling. The experimental data are obviously only available for steady-state conditions, and so the effects of varying conditions during startup or shutdown of the components can be described by modelling. Modelling creep deformation is usually based on the so-called "creep constitutive equations", which describe the strain rate dependence on stress, temperature and time, or creep strain. Unfortunately, the equations are derived from conventional creep experiments under constant load conditions, and so the transient effects upon stress changes are ignored. Because the stress is transformed between model elements, a correct description of the creep behaviour must contain the transient effects. In this work, the conventional approach to describing primary and secondary creep stages is combined with an internal stress model of the transient creep stage to address the problem of the stress changes, as well as that of the low-stress creep regime.

## 1. Introduction

The mathematical modelling of creep is mostly carried out using phenomenological models. Some of these models are just simple curve fitting, whereas others have a more physical meaning. Because the uniaxial creep curve is not easy to describe by one simple relationship, different constitutive laws are usually used to describe primary, secondary, and tertiary creep Menon (1992). Researchers have tended to focus on creep life analysis using numerous extrapolation techniques (Cui et al., 2015; Seruga and Nagode, 2015; Evans, 2010) and by creating continuum mechanics damage models (Tvergaard, 1987; Vandergiessen and Tvergaard, 1994), whereas little effort is spent on identifying and using suitable constitutive models that could describe the transient effects in primary and secondary creep.

Numerical methods such as the finite element method (FEM) are apt tools for incorporating creep constitutive laws to describe the behaviour of real components under steady-state or transient conditions. They can also be used to model atypical or miniature test setups with more complicated geometry such as bending, small punch, or indentation tests and serve for inverse analysis approaches by these test techniques. FEM modelling of deformation and degradation processes has become an important tool for assessing life-limiting processes in various locations on a component. When components are loaded at high temperatures, creep is one of the most important processes involved. A precise and realistic mathematical description of creep is necessary in order to arrive at an output that corresponds to reality.

Constitutive creep equations found in textbooks (Penny and Marriott, 1971) are usually implemented in FEM software systems. These equations were published a very long time ago and are simplistic and fully empirical, attempting to describe existing experimental creep curves precisely. Nevertheless, there are several reasons why these equations cannot be used for realistic modelling of creep phenomena (Kloc, 2015).

It is well known that, depending on the rate of change, there is a large change in the creep rate as a result of changes in stress. The creep rate then returns slowly to an equilibrium value (Kloc and Sklenicka, 2004). Negative creep rate can be obtained after reducing the macroscopic stress even if the stress level remains positive. Furthermore, internal stress generated during the primary creep stage is considered to be responsible for the effect, but this behaviour is ignored completely with current creep constitutive equations because they were derived to describe only constant stress creep experiments. With

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constant load experiments, the change in the existing stress is too slow to reveal such behaviour. On the other hand, in FEM models, the stress is transformed between model elements relatively fast, even when external conditions remain constant. Moreover, the variable external conditions are of primary interest in FEM modelling.

The creep rate of parts of high-temperature equipment under normal operating conditions is too slow to be investigated experimentally. Accelerated tests at higher stress values and/or temperatures are used instead, and the results are extrapolated to the lower creep rate region. However, there are some low-stress creep experiments (Sklenicka et al., 2003; Kloc et al., 2001) showing that the extrapolation can be misleading, considerably underestimating the deformation rate at low stresses. This is another problem of the constitutive equations, i.e., that low-stress creep is extrapolated from high-stress creep behaviour. Obviously, there are some FEM model elements with low stresses, and when their creep behaviour description is invalid, the whole model behaves incorrectly.

The main goal of this work was to formulate a new creep constitutive equation that deals with these problems more correctly then the existing equations.

## 2. Analysis

For the problem analysis, the creep data of P-91-type creep-resistant steel were used. Extensive results from both high-stress and low-stress regions, as well as from stress-change and constant-structure tests, are available in the literature cited below. The steel exhibits very pronounced transient effects, and it is very often used in high-temperature equipment. Therefore, modelling the creep behaviour of this material is very important and valuable.

### 2.1. Experimental

For the formulation of the model, creep data from conventional constant stress tensile tests in the stress range of 100 MPa to 400 MPa and temperature range from 575 °C to 650 °C (Sklenicka et al., 2003) were used. For the low-stress low-strain-rate region, the helicoid spring technique (Kloc and Marecek, 2009) was employed. In this case, the loading mode is torsion which can be interpreted as a pure shear. The shear stress $\vartheta$ was recalculated to equivalent tensile stress $\sigma$ as $\sigma = \vartheta \cdot \sqrt{3}$ using von Mises criterion. Similarly, the shear strain $\gamma$ was recalculated to equivalent tensile strain $\varepsilon$ as $\varepsilon = \gamma/\sqrt{3}$. Compatibility of the results obtained under the same conditions by both methods was confirmed experimentally. The stress range for the low-stress experiments was from about 1 MPa to 98 MPa at temperatures from 600 °C to 650 °C. All the data were already published (Kloc and Sklenicka, 1997; Sklenicka et al., 2003). Transition stages were observed mainly in the low-stress long term experiment with stress changes in the range 34–44 MPa at 600 °C (Kloc and Sklenicka, 2004). The constant structure creep results at 650 °C from the literature (Milička and Dobeš, 1998) were also used.

The primary and secondary stage parts of all creep curves were fitted by the Li equation (Li, 1963)

$$\varepsilon = \dot{\varepsilon}_s t + \dot{\varepsilon}_s \tau \ln\left(1 + \frac{\dot{\varepsilon}_i - \dot{\varepsilon}_s}{\dot{\varepsilon}_s}\left(1 - \exp\left(\frac{-t}{\tau}\right)\right)\right) \tag{1}$$

where $\varepsilon$ is the creep strain, $\dot{\varepsilon}_s$ is the secondary stage creep rate, $\dot{\varepsilon}_i$ is the initial creep rate, $\tau$ is the primary stage relaxation time and $t$ is time. The $\dot{\varepsilon}_s$, $\dot{\varepsilon}_i$ and $\tau$ values were obtained as parameters from the fitting procedure and subsequently used for the creep curves analysis. The primary transient strain $\varepsilon_p$ can be derived as $\varepsilon_p = \dot{\varepsilon}_s \tau \ln(\dot{\varepsilon}_i/\dot{\varepsilon}_s)$.

### 2.2. Low-stress creep

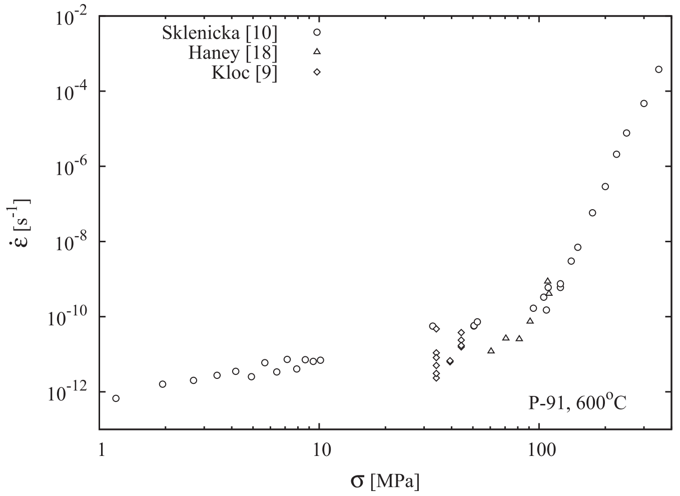

**Fig. 1.** Creep-rate dependence on stress for the P-91 steel at 600 °C.

The question of whether creep behaviour at low stresses is different from that at higher stresses has been disputed for a long time (Kassner et al., 2007; Kumar et al., 2009) without any clear conclusion. The main reason why there is no clear conclusion is that the analysis is almost exclusively based on the stress dependence of the minimum creep rate. This situation is illustrated in Fig. 1. The problem is that while the data points at high stresses represent the measured minimum creep rate, $\dot{\varepsilon}_m$, no tertiary stage, and thus no minimum rate, is detected with low-stress experiments. The creep rate, $\dot{\varepsilon}_e$, after a given time is used instead, and so the values are not fully comparable. The time to estimate $\dot{\varepsilon}_e$ was adjusted to be about $3\tau$, where the $\dot{\varepsilon}_e$ value should not be more than 5% higher than $\dot{\varepsilon}_s$ (see Eq. (1)). It had been assumed that the $\dot{\varepsilon}_e$ values are close to the $\dot{\varepsilon}_m$ value because the ratio of the initial rate to final rate, $\dot{\varepsilon}_i/\dot{\varepsilon}_e$, was found to be the same or even higher than $\dot{\varepsilon}_i/\dot{\varepsilon}_m$ for higher stresses. Later experiments (Kloc and Sklenicka, 2004; Haney et al., 2009) showed that this assumption is not correct; the results of these later experiments are also included in Fig. 1.

To solve this problem, it is necessary to look at the primary-stage creep parameters instead of the secondary-stage ones to obtain comparable values for both the low-stress and high-stress creep regimes. The dependence of the initial creep rate, $\dot{\varepsilon}_i$, on stress is plotted in Fig. 2. It is clear that there is a difference between the low-stress and high-stress regions, despite the relatively large scatter of values. The dependence of the primary-stage relaxation time on stress also clearly shows two distinct regions, as can be seen in Fig. 3, which was taken from a previous paper (Kloc and Sklenicka, 2009). Different values for the apparent activation energy, as demonstrated in Kloc et al. (2001), also indicate that the low-stress creep deformation mechanism is

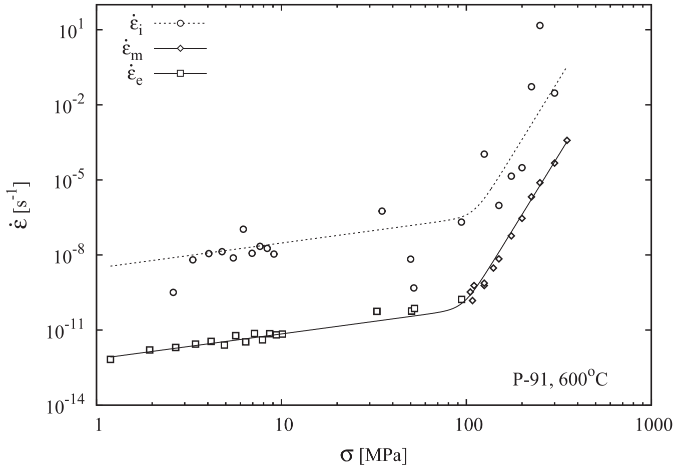

**Fig. 2.** Initial creep-rate dependence on stress for the P-91 steel at 600 °C.