Mechanics of Materials 119 (2018) 92–108

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

On the homogenization of hexagonal honeycombs under axial and shear loading. Part II: Comparison of free skin and rigid skin effects on effective core properties

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ARTICLE INFO

Article history:
Received 22 November 2016
Revised 29 June 2017
Available online 13 September 2017

Keywords:
Honeycombs
Effective elastic properties
Finite element analysis
Periodic boundary conditions
Representative volume element
Free skin and rigid skin effects

ABSTRACT

The second part of the paper is dedicated to the presentation of some analytical formulae used to consider the rigid skin effect. Special attention is given to the lower and upper bounds of out-of-plane shear moduli for a *generalized Malek and Gibson* beam model and to a *correction of Gibson and Ashby* model. The imposed boundary conditions on a honeycomb cell presented in the first part of this research are analyzed separately for the understanding of free and rigid skin effects. Then the finite element method is used to determine the effective elastic properties of the honeycomb cell and a comparison with existing and proposed analytical models is done. The numerical study considers both the free and rigid skin effects for two different finite elements types, Brick and Shell. The influence of including/neglecting the skin effect upon the effective elastic properties is presented and discussed. The finite element modeling (FEM) is performed by using APDL in ANSYS and a good agreement has been achieved between the results of the present analytical models and 3D FEM. Studies of convergence analyses by using Brick and Shell elements are performed for three sets of relative densities as a function of the mesh refinement for the conventional, over-expanded, and re-entrant honeycombs. Some particularities in modeling with Shell elements are discussed by considering the number of layers and the section offset. A sensitivity analysis in terms of the geometric parameters of the unit cell has been conducted for analyzing the core thickness effect. The results provide new insights into understanding the limitations of present analytical models and opens perspectives for the detailed design of commercial honeycombs.

1. Introduction

Several analytical relations for establishing the effective mechanical properties estimations are considered in the literature (e.g. Kelsey et al., 1958; Gibson and Ashby, 1997; Balawi and Abot, 2008a). Gibson and Ashby summarized the analytical formulae for relative density and the in-plane and out-of-plane properties of some honeycombs. The considered hexagonal honeycombs are the *classical* ones, in which the thickness of the all walls is constant, but also for hexagonal honeycombs with double walls attached by gluing along the ribbon direction, which are called sometimes *commercial* honeycombs. All these formulae are obtained principally by using the beam model theory. Currently, analytical models of elements composing honeycombs assemblies are being refined to include axial and shear deformations, in addition to bending deformations, on which all first studies have been based.

All these mathematical models are for pure cellular structures and the presence of the face sheet is not considered. As a result, the existing analytical solutions do not agree well with experimental results (Shi and Tong, 1995) and, clearly, for improving the results the skins of the sandwich, in which the core is a component, must be taken into account.

Considering rigid skin effects, Becker (1998, 2000) obtained the closed-form solution for the in-plane stiffnesses of hexagonal honeycombs depending on the core thickness. Then, Hohe and Becker (2001) derived a closed-form solution to predict components of the effective elasticity tensor and compared them with FE results. Xu and Qiao (2002) studied the thickness effect for evaluation of all effective elastic constants using a multi-pass homogenization technique. In the paper of Chen and Davalos (2005) analytical models which include the skin effect were proposed. Besides the calculus of some effective elastic properties it is possible to determine the stresses at the interface between the honeycomb and the skins considered as rigid. They clearly describe the *warping effect* (in between the top and bottom skins) which has implications at the level of a honeycomb cell. This concept is to be found

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http://dx.doi.org/10.1016/j.mechmat.2017.09.004
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in literature with different names. Grediac (1993) named it *bending effect*, Becker (1998) as *thickness effect*, and Xu and Qiao (2002) as *skin effect*.

A comprehensive review dedicated to the theoretical determinations of the effective stress-strain material behavior of two dimensional cellular materials with large scale cells is presented in Hohe and Becker (2002). According to this work, there are three basic homogenization techniques: surface average based approaches, volume average based approaches and two-scale expansion of the mechanical fields.

The advantages of the finite element (FE) approach for the homogenization analysis were firstly pointed out by Grediac (1993), who analyzed the out-of-plane behavior of the honeycomb structure. This FE approach was applied and extended for calculation for all out-of-plane elastic moduli for the honeycomb and tubular cores by Meraghni et al. (1999). An improved analytical equation for calculation of the equivalent in-plane moduli accounting for the core height but neglecting the skin effect is proposed by Chen and Ozaki (2009a) and Chen et al. (2009b). Lira et al. (2009), extended the analytical method and FE approach, developed for the evaluation of transverse shear elastic properties of hexagonal honeycombs, onto novel multi re-entrant honeycombs. Burlayenko and Sadowski (2010) calculated also the material constants of foam-filled honeycomb cores.

FEM was used both for the verification of the analytical equations and for the direct calculation of the effective elastic properties. Researchers used ANSYS, as Burton and Noor (1997), Scarpa and Tomlin (2000), Xu et al. (2001), Xu and Qiao (2002), Whitty et al. (2002), Balawi and Abot (2008b), Lira et al. (2009), Abbadi et al. (2009), Vigliotti and Pasini (2012), Catapano and Montemurro (2014a). ABAQUS was preferred by Becker (2000), Ju and Summers (2011), Chen and Davalos (2005), Burlayenko and Sadowski (2010), Miller et al. (2011), Li et al. (2015), Malek and Gibson (2015). COSMOS was the preference for Penado (2013), and MSC.MARK for Chen et al. (2008), Chen and Ozaki (2009a), Chen et al. (2009b). Other softwares as: SDRC I-DEAS received the attention of Whitty et al. (2002) and COMSOL of Wang et al. (2015); in some cases were used in-house codes such as MOSAIC by Meraghni et al. (1999), CASTEM 2003 by Abbadi et al. (2009), and the Cast3M-CEA - NIDACORE software by Gornet et al. (2006).

Most of the researchers used shell elements with 4 nodes, named hereby Shell4; out of these we mention: Grediac (1993), Shi and Tong (1995), Meraghni et al., (1999), Becker (2000), Scarpa and Tomlin (2000), Chen and Davalos (2005), Chen et al. (2008), Pan et al. (2008), Chen et al. (2009b), Penado (2013) and Li et al. (2015). Others used shell elements with 8 nodes, named Shell8 as: Burton and Noor (1997), Xu et al. (2001), Xu and Qiao (2002), Burlayenko and Sadowski (2010), and Penado (2013). Solid elements, usually hexahedral with 8 nodes (Brick8) were used by Guedes and Kikuchi (1990), Lira et al. (2009), Malek and Gibson (2015) and Li et al. (2015); also hexahedrals with 20 nodes (Brick20) were used by Gornet et al. (2006), Catapano and Montemurro (2014a,b), Miller et al. (2011) and Burlayenko and Sadowski (2010). In some papers Beam elements are also used as in: Ju and Summers (2011), Whitty et al. (2002) and Balawi and Abot (2008a,b), or even plane 2D finite elements (Vigliotti and Pasini, 2012). Almost all the finite elements used for honeycomb modeling are of *h* class, that is for increasing the precision the dimensions of the elements are reduced, but in a few papers an adaptive finite element method *p* is introduced in order to improve the accuracy of the numerical results, for example Guedes and Kikuchi (1990).

Considering the skin effect, Grediac (1993) applied the FE method to study core cells with different core configurations and obtained a correction formula for the longitudinal shear modulus, and also studied the stress distribution in core walls. He concluded that the skin effect is a localized phenomenon limited only to the region adjacent to the interface. Burton and Noor (1997) used detailed FE models to examine the effect of the adhesive joint on the load transfer and static responses of sandwich panels. The first paper on equivalent in-plane moduli considering skin effect was the work by Becker (1998). He derived a closed-form solution to predict the in-plane moduli and compared them with FE results. A further expansion was attempted by Hohe and Becker (2001) to include all stiffness components for general honeycomb cores, but same as in the first paper of Becker, by using implicit calculations and a pre-defined hyperbolic cosine function to describe the displacements. Xu and Qiao (2002) applied a multi-pass homogenization method to study the stiffness for transverse shear, in-plane stretch and out-of-plane bending. In these studies, the inclined panel was unfolded into the plane of flat panel, and therefore, the solution corresponds to a 2-D model. Chen and Davalos (2005) developed a method based on basic equilibrium equations to calculate the stiffness as well as the interfacial stresses for in-plane and out-of-plane effective elastic properties. Chen et al. (2008) pointed out that there are two error factors to apply the rule of mixture to a honeycomb sandwich, the first one because the inclined cell wall deforms more than the vertical cell wall, and the second one, due to the interference (or warping) with the face. An analytic homogenization method, using trigonometric function series was recently proposed by Li et al. (2015) to study the influence of the honeycomb height on the elastic in-pane properties, and the upper and lower bounds of the equivalent elastic moduli. The interfacial stresses were also studied. Numerical homogenized models were established for the stretching, bending and the stretch–bending coupled problem, respectively.

Catapano and Montemurro (2014a,b), affirmed that a common weakness of the works about FE-based homogenization techniques consist in the use of shell-like models for the unit cell of the honeycomb core because these models do not take into account the true geometry of the cell and, consequently, they are not able to properly estimate the influence of the real 3D stress state on the effective core properties. However, some of the published papers present results obtained with shell models which were also verified experimentally.

Under these circumstances, the present paper is analyzing the conditions in which the shell elements can lead to correct results. A thorough presentation of the most important papers in the domain was done by Malek and Gibson (2015), and in the same paper more accurate formulae of all nine elastic constants are obtained by modifying the classic analysis to account for the nodes at the intersection of the vertical and inclined members. Still, in this paper the analytical approach for classic and commercial honeycombs is using different geometric parameterization. Starting from the same idea of decomposing the cell in independent members, the present paper is proposing a unified model, valid for any type of honeycomb, and therefore generalizing the model proposed by Malek and Gibson. In the same time, a simplified model proposed in this paper is based on the inclusion of the node of intersection of the walls in equivalent parallelepiped members.

For the verification of such resulting analytical relations, and for analyzing the differences which are obtained by neglecting and considering the skin effect, FEMs are used. In this paper, the same RVE, i.e. 1/8 of the repetitive cell is considered for all analyses. Still, in the phase of developing the methodology of implementation, the repetitive (whole) cell is used as to let better understand several aspects developed in this paper.

This paper is a continuation of Part I in which the free skin analysis and specific equations are presented. Also comparisons by considering the free and rigid effects are done through analytical relations and numerical formulations by using FEA. Therefore, in Chapter 2 a *generalized Malek and Gibson* beam model considering a unified system of establishing equivalent lengths of cell walls