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Research paper

A pileup of edge dislocations against an inclined bimetallic interface

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ABSTRACT

A pileup of edge dislocations against an arbitrarily inclined flat bimetallic interface is considered. Equilibrium positions of dislocations are determined for a given number of dislocations and specified material properties, assuming that the resolved shear stress along the pileup plane from a remotely applied loading is uniform and equal for all interface inclination angles. Numerical results are compared for pileups at 0°, 30°, 45°, and 60° relative to the interface normal. The overall dislocation distribution is mildly affected by the inclination of the interface, although there are some notable differences. While an inclined interface repels the first and last dislocation stronger than the orthogonal interface, for piled-up dislocations in-between this is not necessarily the case. Small differences in the pileup length and the proximity of the leading dislocation to differently inclined interfaces can considerably affect the interface stresses. The magnitude of interface stresses decreases with the increase of the shear moduli ratio G_2/G_1 due to stronger repulsion exerted on dislocations by stiffer interfaces. The disparity in Poisson's ratio also affects the interface stresses. The back stress behind a trailing dislocation is evaluated and discussed.

1. Introduction

The study of dislocation pileups against second-phase particles and grain boundaries has been a classical topic of mechanics and materials science of importance for the analysis of inelastic material response and fracture. An early study of dislocation pileups was performed by Eshelby et al. (1951), who considered pileups in an infinite homogeneous medium in which the leading dislocation was assumed to be locked. Further contributions were made by many investigators, who addressed pileups of screw and edge dislocations against circular inhomogeneities and bimetallic interfaces. The effects of elastic anisotropy and the nonlinearity due to dislocation cores were examined, as well as the stress fields of double ended pileups in stacked slip planes and of multiple dislocation-wall pileups (Chou, 1966; Barnett and Tetelman, 1967; Barnett, 1967; Kuang and Mura, 1968; Thölén, 1970; Smith, 1972; Kuan and Hirth, 1976; Wagoner, 1981; Öveçoğlu et al., 1987; Voskoboinikov et al., 2007; 2009; Hall, 2010; Baskaran et al., 2010; Geers et al., 2013; Scardia et al., 2014; Zhang, 2017; Kapoor and Verdhan, 2017).

Edge dislocation pileups against a plane bimetallic interface were studied analytically by the method of continuously distributed infinitesimal dislocations by Kuang and Mura (1968). They solved analytically the singular integral equation for equilibrium positions of dislocations, but their solution involved infinite products which were quite demanding for computations. Discrete edge dislocation pileups were investigated numerically by Kuan and Hirth (1976), who incorporated in their analysis the nonlinear dislocation core terms. Wagoner (1981) studied the corresponding anisotropic elastic effects. Öveçoğlu et al. (1987) also considered discrete edge dislocation pileups against a plane bimetallic interface, and evaluated the interface stresses for various combinations of material parameters. More recently, Voskoboinikov et al. (2009) presented an asymptotic analysis of dislocation pileups against a bimetallic interface, while Lubarda (2017a) presented an analysis of dislocation pileups against both a circular inhomogeneity and a flat bimetallic interface.

In all of the above work the dislocation pileups were assumed to be along the glide direction orthogonal to the interface. In the present paper we extend these analyzes by considering discrete edge dislocation pileups against a flat bimetallic interface which is arbitrarily oriented relative to the pileup (glide) direction. We solve numerically the nonlinear algebraic equations that specify equilibrium positions of dislocations, for a given number of dislocations and specified material properties. The magnitude of the resolved shear stress along the pileup direction is assumed to be uniform and equal for each inclination φ of the interface. The derivation of the expressions for dislocation forces, which must vanish in equilibrium, is lengthy and tedious, but we were able to cast them in a relatively compact form for any angle φ. The simplified expressions are then deduced for $\varphi = 0°$, 30°, 45°, and 60°. The overall dislocation distribution is mildly affected by the inclination of the interface, although there are some notable differences. While an inclined interface repels the first and last dislocation stronger than the interface orthogonal to the glide plane, for piled-up dislocations in-

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between this is not necessarily the case. Furthermore, small differences in the pileup length and the proximity of the leading dislocation to differently inclined interfaces can considerably affect the interface stresses. The magnitude of these stresses decreases with the increase of the shear moduli ratio G_2/G_1 due to stronger repulsion exerted on dislocations by stiffer interfaces. The disparity in Poisson's ratio also affects the interface stresses. The variation of the back stress behind a trailing dislocation of a pileup is determined for different orientation of the interface. There is a small effect of φ on the magnitude of the back stress. Far behind a trailing dislocation, the back stress approaches the stress levels caused by a superdislocation of the Burgers vector Nb located at the interface, independently of φ. An analysis of screw dislocation pileups against an inclined bimetallic interface is reported in Lubarda (2017b).

In the presented analysis, the dimensionless material parameters α and β (Dundurs, 1969) are used, which are defined in terms of $\Gamma = G_2/G_1$ and (ν_1, ν_2) by

$$\alpha = \frac{(1-\nu_1)\Gamma - (1-\nu_2)}{(1-\nu_1)\Gamma + (1-\nu_2)}, \quad 2\beta = \frac{(1-2\nu_1)\Gamma - (1-2\nu_2)}{(1-\nu_1)\Gamma + (1-\nu_2)}. \tag{1}$$

After nondimensionalisation of the problem, the two dimensionless parameters that play a prominent role for the most part of the analysis are the parameters q and γ, defined in terms of α and β by

$$q = \frac{\alpha - \beta}{1+\beta}, \quad g = \frac{(1+\alpha)\beta}{1-\beta^2}, \quad \gamma = q + g = \frac{\alpha + \beta^2}{1-\beta^2}. \tag{2}$$

For example, if $\Gamma = \infty$ (rigid interface, $G_2 \gg G_1$), the dimensionless parameters are

$$\alpha = 1, \quad 2\beta = \frac{1-2\nu_1}{1-\nu_1}, \quad q = \frac{1}{3-4\nu_1}, \quad (1/3 \le q \le 1), \quad 2\gamma = q + \frac{1}{q}, \quad (1 \le \gamma \le 5/3).$$

If $\Gamma = 1$ (equal shear moduli of two materials, $G_1 = G_2$), then

$$\alpha = \beta = \frac{\nu_2 - \nu_1}{2 - (\nu_1 + \nu_2)}, \quad q = 0, \quad 2\gamma = \frac{\nu_2 - \nu_1}{1 - (\nu_1 + \nu_2)}.$$

2. Edge dislocation pileups

Fig. 1 shows a pileup of N positive edge dislocations of a Burgers vector $b_x = b > 0$ against a bimetallic interface inclined by an angle

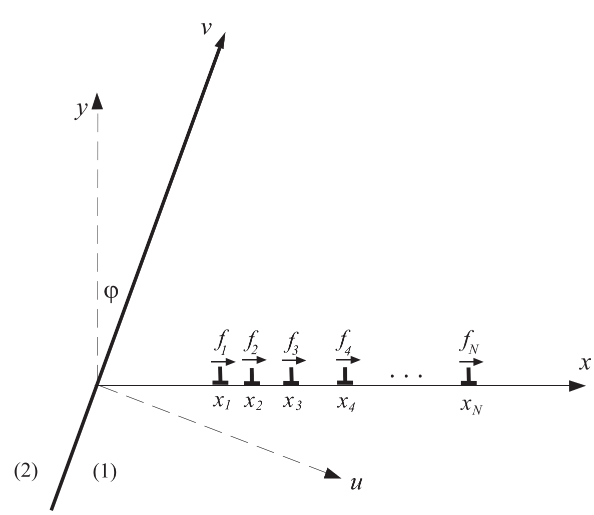

Fig. 1. A pileup of N edge dislocations against a bimetallic interface under uniform resolved shear stress along the glide plane $y = 0$. A bimetallic interface ($u = 0$) is inclined by an angle φ relative to the glide plane normal y. In the equilibrium configuration the configurational force on each dislocation vanishes ($f_i = 0$), which specifies the corresponding positions of dislocations x_i ($i = 1, 2, 3, \cdots, N$).

$\varphi \neq \pm 90^\circ$ relative to the glide plane normal. It is assumed that the resolved shear stress along the glide plane of dislocations, caused by remotely applied loading, is uniform along the glide plane and independent of φ. We denote this shear stress by τ^a. It is the only part of applied stress that is relevant to dislocation motion (glide) considered in this paper. The external loading has to be such that τ^a is directed toward the interface, i.e., $\tau^a_{xy}|_{y=0} = -\tau^a$, in order to have a driving force for piling-up of dislocations in the case $\Gamma > 1$. The contribution to the resolved shear stress from the interactions among dislocations in the presence of the interface can be calculated by using the results of Head (1953) and Dundurs (1969), as described in the appendix of the paper. For the ith dislocation, at the position x_i, this shear stress is

$$\tau^b(x_i) = k_1 b \frac{\gamma}{2x_i} + \sum_{j \neq i}^{N} \tau^{b,j}(x_i), \quad k_1 = \frac{G_1}{2\pi(1-\nu_1)}. \tag{3}$$

The first term on the right hand-side is the self-shear stress contribution of the dislocation at x_i, due to its image effects caused by the interface, while the second term is the sum of resolved shear stresses caused by all other dislocations in the pileup. In the equilibrium pileup configuration, the glide component of dislocation force on each dislocation must vanish,

$$f_i(x_i) = [\tau^b(x_i) - \tau^a]b = k_1 b^2 \frac{\gamma}{2x_i} + b\sum_{j \neq i}^{N} \tau^{b,j}(x_i) - \tau^a b = 0, \quad (i = 1, 2, 3, \cdots, N). \tag{4}$$

The expressions for $\tau^{b,\,j}(x_i)$ are derived in the appendix of this paper for any angle $\varphi \neq \pm 90^\circ$. In particular, if the interface is orthogonal to glide plane ($\varphi = 0^\circ$), the resolved shear stress $\tau^{b,\,j}(x_i)$ takes an explicit, compact form

$$\tau^{b,j}(x_i) = k_1 b \left[\frac{1}{x_i - x_j} + \gamma \frac{1}{x_i + x_j} + 2q \frac{x_j(x_i - x_j)}{(x_i + x_j)^3} \right]. \tag{5}$$

For $\varphi = 30^\circ$, the expression is

$$\tau^{b,j}(x_i) = k_1 b \left[\frac{1}{x_i - x_j} + \frac{\gamma}{2} \frac{2x_i + x_j}{x_i^2 + x_j^2 + x_i x_j} + \frac{9q}{2} \frac{x_i x_j^2 (x_i^2 - x_j^2)}{(x_i^2 + x_j^2 + x_i x_j)^3} \right], \tag{6}$$

while for $\varphi = 45^\circ$,

$$\tau^{b,j}(x_i) = k_1 b \left[\frac{1}{x_i - x_j} + \gamma \frac{x_i}{x_i^2 + x_j^2} - q \frac{x_j (x_i^2 - x_j^2)(x_i^2 + x_j^2 - 4x_i x_j)}{(x_i^2 + x_j^2)^3} \right]. \tag{7}$$

Finally, for $\varphi = 60^\circ$ the resolved shear stress is found to be

$$\tau^{b,j}(x_i) = k_1 b \left[\frac{1}{x_i - x_j} + \frac{\gamma}{2} \frac{2x_i - x_j}{x_i^2 + x_j^2 - x_i x_j} - \frac{q}{2} \frac{x_j (x_i^2 - x_j^2)(2x_i^2 + 2x_j^2 - 5x_i x_j)}{(x_i^2 + x_j^2 - x_i x_j)^3} \right]. \tag{8}$$

In the limiting case $\varphi = 90^\circ$, a pileup is along the interface, made of N interface edge dislocations, provided that the leading dislocation in the pileup is locked. In this case, the interaction shear stress is

$$\tau^{b,j}(x_i) = (1+\gamma) k_1 b^2 \left(\frac{1}{x_i} + \sum_{\substack{j=2 \\ j \neq i}}^{N} \frac{1}{x_i - x_j} \right), \quad (i = 2, 3, \cdots N), \tag{9}$$

where x is measured from the position of the locked dislocation ($x_1 = 0$), and $1 + \gamma = (1+\alpha)/(1-\beta^2)$. From the physical point of view, pileups of interface dislocations are of less significance, albeit they may be of some interest in the analysis of semicoherent interfaces, for which interface dislocations play important role in misfit accommodation and strain relaxation (Freund, 1993; Lubarda and Kouris, 1996; Lubarda, 1998).