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Stability analysis of the phase-field method for fracture with a general degradation function and plasticity induced crack generation

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ABSTRACT

The phase-field method is a popular technique for modeling crack initiation, bifurcation and coalescence without the need to explicitly track the crack surfaces. In this framework, the governing equations are derived from thermodynamic principles and cracks are modeled as continuous entities, whose width is defined by a small process zone parameter.

Propagation of cracks is governed by the partition of strain energy that contributes to fracture and may be stable, in which case additional energy is required to form cracks, or unstable, where cracks advance with no additional input energy.

In this work we propose a stability framework, based on a linear perturbation analysis, to determine the onset of unstable crack growth. The derivations lead to an analytical, energy based criterion for the phase field method in linear elastic and visco-plastic materials. The stability criterion is valid for a general degradation function and accounts for fracture induced by cold-work.

Numerical results on linear elastic materials show that the proposed criterion not only recovers the critical stress value reported in the literature but also provides a stability criterion for visco-plastic materials with a general degradation function. The criterion is tested on one dimensional problems and a two dimensions homogeneous example, successfully predicting the instability point.

1. Introduction

The phase-field method has been extensively used to model phase transformations in materials, particularly when dealing with problems with moving boundaries. These phases can be associated with several distinct physical phenomena, that include solidification Beckermann et al. (1999); Boettinger et al. (2002); Jeong et al. (2001), solid-state phase transformation (Wen et al., 2000; Zaeem and Mesarovic, 2010), martensitic transformation (Wang and Khachaturyan, 1997; Mamivand et al., 2013), dislocation dynamics (Wang et al., 2001), grain growth (Fan and Chen, 1997; Kazaryan et al., 2000), among others. In this work we use the phase-field formulation within the scope of fracture mechanics, which has been extensively studied (Amor et al., 2009; Borden et al., 2012; Bourdin et al., 2000; Duda et al., 2015; Kuhn and Müller, 2013; May et al., 2015; Miehe et al., 2010a; 2010b; Vignollet et al., 2014; Karma and Lobkovsky, 2004; Aranson et al., 2000; Karma et al., 2001; Eastgate et al., 2002).

The phase-field formulation for fracture, approximates cracks as continuous entities and tracks their evolution in the domain. In this framework, a surface density function, which depends on a process zone parameter l_0 (Bourdin et al., 2000; Klinsmann et al., 2015) is used to represent the crack, as depicted in Fig. 1.

This approach results in a mathematical formulation for a diffused crack that is closely related to gradient damage mechanics (Pham and Marigo, 2011; Voyiadjis and Mozaffari, 2013), although it has been argued that this correspondence is somewhat coincidental since the approaches focus on modeling different phenomena (Voyiadjis and Mozaffari, 2013; Bourdin et al., 2010). The significant difference between a phase field approach and a discrete crack approach is in the representation of a crack, continuous versus discrete, and would mostly depend on the application problem. Nonetheless, continuous representation of cracks has the advantage that it allows the functional minimization problem to be solved by numerical methods, such as the Finite Element Method, without the need for any special set of shape-functions or enrichments. Hence, complex crack patterns such as branching and coalescence can easily be captured with this approach. Even though this technique has been initially derived to tackle brittle fracture, it has also been employed to model brittle, quasi-brittle and ductile fracture combined with plasticity

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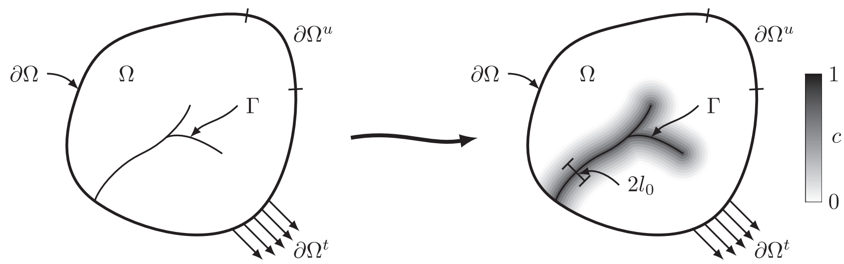

Fig. 1. Schematic depiction of a solid Ω with a discontinuity Γ (crack). In the phase-field formulation the crack is simulated by the field c where a black color corresponds to a fully damaged material ($c = 1$) and a white color corresponds to a fully intact material ($c = 0$). The l_0 parameter controls the width of the process zone.

(Ambati et al., 2015; Duda et al., 2015; McAuliffe and Waisman, 2015; Miehe et al., 2015b). Here quasi-brittle fracture refers to fracture events where the energy supplied for an unstable fracture is dissipated in a localized yielding zone around the crack tip, as described by Bažant (Bažant, 1984) and referred to as "brittle-like fracture in ductile materials" by Rice (Rice, 1966). Rice also argues that an energy-based approach in highly ductile fracture might not be appropriate since some of the hypothesis of Griffith-like fracture breaks down.

In a homogeneous problem (e.g. a bar in tension without imperfections) the phase-field formulation has two distinct behaviors. Initially, the entire problem behaves homogeneously with initiation of damage due to the driving term of the phase-field equation (Miehe et al., 2015a). After a certain critical point, the system loses stability and a non-homogeneous deformation is possible, eventually leading to fracture. Consequently, stability analysis is fundamental to the understanding of crack nucleation within this framework.

More generally, stability analysis is a useful tool in predicting the behavior of boundary value problems. In quasi-static simulations stability can be affected by phenomena like bifurcation due to non-associated flow law (De Borst, 1988; Needleman, 1979; Bigoni, 2000) and will limit the robustness of the numerical approach, often requiring more advanced stepping procedures (e.g. the arc-length method (de Borst, 1987)) and regularization techniques to prevent mesh dependency in localization phenomena (Pham et al., 2011; Abu Al-Rub and Voyiadjis, 2006; Belytschko et al., 1994; Wright and Batra, 1985). In dynamic problems stability is often associated with the appearance of a non-homogeneous type of solution, for example in thermo-mechanical shearband localization (Bai, 1982; Arriaga et al., 2015; 2016).

Therefore, since fracture can be viewed as the localization of damage, stability analysis provides a necessary condition for the onset of localization. In addition, local stability analysis provides relevant information regarding the spatial distribution of localization regions, which in turn allows for the construction of accurate phenomenological models. In addition this technique can be used as a criterion for mesh refinement or injection of strong discontinuities (Rabczuk et al., 2007; Belytschko et al., 2003; Song et al., 2006; Belytschko et al., 2008; Tabarraei et al., 2013; Rabczuk and Samaniego, 2008).

In this work we study the stability behavior of the phase-field formulation applied to linear elastic materials and visco-plastic materials with isotropic hardening. Our formulation is based on a linear perturbation method as opposed to the loss of ellipticity criterion obtained by the well known eigenanalysis of the acoustic tensor, since for a rate-dependent material the equations remain elliptical (Lemonds and Needleman, 1986; Needleman, 1988) and therefore the latter method cannot provide an adequate criterion. The linear perturbation methodology has been extensively used for localization problems, e.g. shear bands (Anand et al., 1987; Bai, 1982; Batra and Wei, 2007; Dai and Bai, 2008), and has proven to lead to reliable criteria.

In this work we restrict our stability analysis to 1D as it is often presented in the literature (Bai, 1982; Bazant et al., 1984; Fressengeas and Molinari, 1987; Batra and Chen, 2001; Batra and Wei, 2007). Nonetheless, our ongoing work intends to extend this analysis to higher dimensions, both analytically and numerically through the technique described in Arriaga et al. (2015).

This work is structured as follows. In Section 2 the governing equations are obtained from thermodynamic considerations and the problem is stated. In Section 3 we derive the stability criterion based on a linear perturbation method for a 1D formulation of the problem. Here we show that this analysis recovers the stability condition developed in the literature for an elastic homogeneous solution and furthermore is expanded to a visco-plastic material with isotropic hardening, a general degradation function and plasticity induced fracture. In Section 4 we present numerical simulations confirming the analytical predictions and in Section 5, the results of the application of the criterion to a 2D homogeneous example is shown. Finally, the conclusions are presented in Section 6.

2. Problem statement

In this section the balance and thermodynamic laws that describe the problem are introduced first, followed by the specification of the free energy function and the internal micro-force, the micro-traction and the macro-traction relations.

These derivations are based on the works of Stumpf and Hackl (2003) and McAuliffe and Waisman (2015), where the physical processes that lead to damage (micro-cracks, micro-voids, dislocations, etc.) correspond to a micro-force balance law. For the analysis, we consider a temperature independent, small-strain formulation within elastic and elastic-viscoplastic material models.

2.1. Balance laws

The macro-force balance law that corresponds to a classical balance of momentum at the continuum scale is given by

$$\rho \ddot{u}_i = \sigma_{ij,j} + b_i \quad (1)$$

with ρ as the density, u_i the displacement field, σ_{ij} the stress tensor and b_i the body force. A superimposed dot ($\dot{x}$) corresponds to a time derivative.

The physical phenomena that drive the damage evolution is described by the so called micro-force balance equation (Stumpf and Hackl, 2003; McAuliffe and Waisman, 2015), given by

$$\rho \theta_c \ddot{c} = H_{i,i} - K + G \quad (2)$$

where c corresponds to the phase-field parameter, θ_c corresponds to the micro-inertia, H_i the micro-traction, K the internal micro-force and G the external micro-force. The phase-field parameter c ranges from 0 to 1, where the value of 0 corresponds to an uncracked state and the value of 1 to a fully cracked state.