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Research paper

Effect of size on necking of dynamically loaded notched bars

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A R T I C L E I N F O

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The influence of material inertia on neck development in a notched round bar is analyzed numerically. Dynamic axisymmetric calculations are carried out for isotropically hardening elastic-viscoplastic solids so that both material strain rate sensitivity and material inertia are accounted for. The focus is on the effect of bar size on whether the notch triggers necking or necking initiates away from the notch. The governing equations are presented in non-dimensional form and two key non-dimensional groups that involve both material and loading parameters are identified. For both non-dimensional groups, with all parameters fixed except for bar size, it is found that for sufficiently small bars, the notch triggers necking, whereas for sufficiently large bars necking ultimately occurs away from the notch. With material properties fixed, for one non-dimensional group the transition to necking away from the notch corresponds to increasing imposed velocity whereas for the other non-dimensional group this transition takes place for decreasing imposed strain rate. Both these transitions correspond to increasing bar size. The results indicate that this transition is governed by material inertia but the bar size at which it occurs depends on the material properties, particularly strain hardening and strain rate hardening.

1. Introduction

There is an extensive, more than 100 year old, literature on the mechanics of necking in the uniaxial tensile test. The classical criterion of Considère (1885) holds for necking of a tensile bar in the limiting case of a infinitely long, thin bar and states that necking initiates at the maximum load. For any finite aspect ratio, there is a delay between the maximum load point and the onset of necking that increases as the bar becomes more stubby, (Needleman, 1972; Hutchinson and Miles, 1974; Hutchinson and Neale, 1977). The literature on the analyses of necking in tensile bars includes one dimensional analyses as well as full three dimensional finite element solutions, involving both quasi-static and dynamic formulations, and analyses that account for effects of various mechanical properties, such as thermal softening, porosity induced softening, bar size and shape, etc. Reviews of tensile bar necking analyses are provided by Hutchinson (1979); Molinari et al. (2014).

For rate independent plasticity and quasi-static deformations, the onset of necking in a uniform circular cylindrical tensile bar is associated with a bifurcation from a state of homogeneous uniaxial tension (Cheng et al., 1971; Needleman, 1972; Hutchinson and Miles, 1974). The bifurcation mode is associated with a sinusoidal variation in the radial dimension of the bar with the longest possible wavelength consistent with the bar length and the boundary conditions at the bar ends. A geometrical imperfection leads to the fairly abrupt development of this mode at an overall strain somewhat less (depending on the imperfection) then the bifurcation strain. Once the neck develops, the classic approximate analysis of Bridgman (1952), and subsequent full numerical solutions, e.g. Chen (1971); Needleman (1972); Argon et al. (1975); Norris et al. (1978); Tvergaard and Needleman (1984); Needleman and Tvergaard (1985), show that the neck curvature induces stress triaxiality that plays a key role in the ductile failure process.

For a viscoplastic solid under quasi-static loading conditions, the onset of necking is no longer associated with a bifurcation. However, the onset of necking can be analyzed as the growth of an initial inhomogeneity (Hutchinson and Neale, 1977). As for a rate independent plastic solid, a notch serves as an imperfection that triggers necking and sets the neck location. Material rate sensitivity leads to a delay in the onset of necking (Hutchinson and Neale, 1977).

In addition, for both rate independent and rate dependent plastic solids characterized by a classic plastic constitutive relation, there is no material length scale in a quasi-static analysis. Hence, the evolution of the neck with strain (at the same imposed strain rate for viscoplastic solids) is independent of specimen size.

The necking behavior under dynamic loading conditions, (e.g. Needleman (1991); Knoche and Needleman (1993); Fressengeas and Molinari (1994); Guduru and Freund (2002); Mercier and Molinari (2003); Rusinek et al. (2005); Osovski et al. (2013); Vaz-Romero et al. (2015); Rotbaum et al. (2015)), can be quite dif-

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ferent than under quasi-static conditions. Material inertia tends to slow neck development, (Needleman, 1991; Xue et al., 2008); multiple necking can occur, (e.g. Knoche and Needleman (1993); Fressengeas and Molinari (1994); Guduru and Freund (2002)); there are size effects, (e.g. Rusinek et al. (2005); Knoche and Needleman (1993)), and neck development can ignore the presence of notches (Rotbaum et al., 2015). Experiments and modeling carried out in Rotbaum et al. (2015) showed that under dynamic loading conditions, the onset of necking in notched tensile bars could occur away from the notch location.

Since material inertia implicitly introduces a length scale, different size specimens deformed at the same strain rate may respond differently. As a consequence, there can be a dependence of the failure strain on specimen size (Knoche and Needleman, 1993). Knoche and Needleman (1993) carried out finite deformation dynamics analyses aimed at modeling the effect of specimen size at a fixed imposed strain rate, on ductile failure in geometrically self-similar tensile bars having various sizes. The material was modeled as a viscoplastic progressively cavitating solid. It was found that the variation of the necking strain with specimen size was not monotonic; the response of sufficiently small specimens was essentially quasi-static and size independent, the failure strain then increased with specimen size before eventually decreasing for sufficiently large specimens.

In this study, a combination of the issues addressed in Knoche and Needleman (1993) and Rotbaum et al. (2015) is considered. In particular, a main focus in this paper is to continue exploring the issue raised by Rotbaum et al. (2015) concerning the circumstances, for dynamic loading conditions, under which necking ignores the presence of a notch.

Calculations are carried out for geometrically similar, dynamically loaded notched circular cylindrical tensile bars of various sizes. Attention is restricted to axisymmetric deformations. The bar material is characterized as an isotropically hardening viscoplastic Mises solid. A non-dimensional form of the governing equations is presented and two key non-dimensional ratios are identified: one relates the bar length to a characteristic length that depends on material properties and the imposed velocity, while the other relates the imposed strain rate (the imposed velocity divided by the bar length) to a material characteristic strain rate. Both of these non-dimensional ratios involve the bar length. The focus of the results is on the transition from necking at the notch cross section to necking away from the notch cross section as the specimen size is varied.

2. Problem formulation

As in Knoche and Needleman (1993), the calculations are based on a convected coordinate Lagrangian formulation of the field equations. The independent variables are taken to be the particle positions in the initial stress free configuration of the axisymmetric tensile bar and time. In the current configuration the material point initially at $\mathbf{X}$ is at $\mathbf{x}$. The displacement vector $\mathbf{u}$ and the deformation gradient $\mathbf{F}$ are defined by

$$\mathbf{u} = \mathbf{x} - \mathbf{X}, \quad \mathbf{F} = \frac{\partial \mathbf{u}}{\partial \mathbf{X}} \tag{1}$$

The principle of virtual work accounting for material inertia is written as

$$\int_V \mathbf{S} : \delta \mathbf{F} dV = \int_B (\mathbf{S} \cdot \mathbf{n}) \cdot \delta \mathbf{u} dB - \int_V \rho \frac{\partial^2 \mathbf{u}}{\partial T^2} \cdot \delta \mathbf{u} dV \tag{2}$$

Here, T is time, $\mathbf{S}$ is the (unsymmetric) nominal stress tensor, $\mathbf{S} = (\det \mathbf{F}) \mathbf{F}^{-1} \cdot \boldsymbol{\Sigma}$ with $\boldsymbol{\Sigma}$ the Cauchy stress, ρ is the mass density, and V and B are, respectively, the volume and the surface of the body in the undeformed reference configuration.

Attention is confined to axisymmetric deformations and, for notational simplicity, we use r and z to denote the convected Lagrangian coordinates. The initial length of the bar is $2L_0$ and the initial radius, which varies along the bar is denoted by $R_0(z)$. The bar occupies the region $-L_0 \le z \le L_0$, $0 \le r \le R_0(z)$.

An axial velocity $V(t)$ is imposed on $z = L_0$ together with symmetry about $z = 0$. This means that the loading is actually applied at $z = -L_0$ as well. The reason for imposing symmetry about $z = 0$ is that without this symmetry and with shear free conditions on the loading ends, the preferred quasi-static necking mode would be the long wavelength mode with the neck forming at one of the ends. Thus, under quasi-static loading conditions the deformation and stress concentrations associated with the centrally placed notch would be competing with those associated with the preferred necking mode. On the other hand, if shear constraints were imposed at the ends, then the onset of necking would be affected by both the notch and the deformation gradient imposed by the constraints, complicating the interpretation of the notch effect. With symmetry about $z = 0$ imposed, the preferred quasi-static necking mode is driven by the presence of the notch and necking occurs at the notch cross section. The occurrence of necking away from the notch cross section, when it occurs, is a dynamic effect.

The boundary conditions on the region analyzed are $u_z(r, L_0, T) = V(T)$ where

$$V(T) = \begin{cases} V_1 T/T_r, & \text{for } T \le T_r \\ V_1, & \text{for } T > T_r \end{cases} \tag{3}$$

Here, V_1 is the magnitude of the imposed velocity and T_r is the rise time.

The other displacement boundary conditions imposed are $u_z(r, 0, T) = 0$ and $u_r(0, z, T) = 0$. All other boundary conditions correspond to zero imposed tractions.

The material is characterized as an elastic-viscoplastic Mises solid. The total rate of deformation, $\mathbf{D} = \mathrm{sym}(\dot{\mathbf{F}} \cdot \mathbf{F}^{-1}$ with sym( ) denoting the symmetric part, is written as the sum of an elastic (actually hypoelastic) part, $\mathbf{D}^e$, and a viscoplastic part, $\mathbf{D}^p$, with

$$\mathbf{D}^e = \frac{1+\nu}{E} \tilde{\boldsymbol{T}} - \frac{\nu}{E} \,\mathrm{tr}(\tilde{\boldsymbol{T}}) \mathbf{I} \tag{4}$$

where E is Young's modulus, ν is Poisson's ratio, $\boldsymbol{T} = (\det \mathbf{F}) \boldsymbol{\Sigma}$, $(\tilde{\ })$ denotes the Jaumann rate based on T, tr( ) denotes the trace and $\mathbf{I}$ is the identity tensor.

The viscoplastic flow rule is

$$\mathbf{D}^p = \frac{3 \dot{\Lambda}^p}{2 \Sigma_e} \boldsymbol{T}' \tag{5}$$

where $\mathbf{I}$ is the identity tensor, $\dot{\Lambda}^p$ is the effective plastic strain rate, and the Kirchhoff stress deviator $\boldsymbol{T}'$ and effective stress Σ_e are given by

$$\boldsymbol{T}' = \boldsymbol{T} - \Sigma_h \mathbf{I}, \quad \Sigma_e = \sqrt{\frac{3}{2} \boldsymbol{T}' : \boldsymbol{T}'}, \quad \Sigma_h = \frac{1}{3} \,\mathrm{tr}(\boldsymbol{T}) \mathbf{I} \tag{6}$$

The material response, the bar size and shape, and the boundary value problem are characterized by a collection of non-dimensional quantities. To put the equations in non-dimensional form, we normalize all stress quantities by a reference stress σ_0, all length quantities by a reference length L_c and all time quantities by a reference time t_c.

The principle of virtual work, Eq. (2), can be written as

$$\int_v \mathbf{s} : \delta \mathbf{F} dv = \int_b (\mathbf{s} \cdot \mathbf{n}) \cdot \delta \mathbf{w} db - \int_V \ddot{\mathbf{w}} \cdot \delta \mathbf{w} dv \tag{7}$$

provided

$$t_c = L_c \sqrt{\frac{\rho}{\sigma_0}} \tag{8}$$

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