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Towards designing composites with stochastic composition: Effect of fluctuations in local material properties

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ABSTRACT

This article presents a numerical study of the mechanical behavior of particulate composites with stochastic composition. Two types of such materials are considered: composites with homogeneous elastic–plastic matrix and randomly distributed inclusions of stiffness sampled from a distribution function, and composites with matrix having spatially varying elastic–plastic material parameters with no inclusions as well as with randomly distributed identical inclusions. We observe that the presence of fluctuations in either inclusions or matrix material properties leads to smaller effective modulus, smaller strain hardening and a reduction of the yield stress of the composite. Fluctuations of the yield stress of the matrix leads to a significant reduction of the mean yield stress of the composite. Fluctuations of the elastic modulus and of the strain hardening are associated with the reduction of the mean of the distributions of elastic modulus and strain hardening of the composite. For the range of parameters considered, fluctuations lead to maximum principal stress fields with narrow distribution of values, which implies enhanced resistance to damage initiation. Increasing the variance of the distribution functions from which local material properties are sampled, while keeping the mean constant, renders these effects more pronounced. This study is motivated by the growing interest in additive manufacturing technologies which open new possibilities for designing composite materials.

1. Introduction

Composite materials are broadly used in engineering. The typical composite is made from two constituents, the matrix and the filler, with the filler distribution being either random or controlled. Examples are particulate composites with randomly distributed reinforcing particles, fiber composites with short, chopped fibers randomly distributed in the matrix, and woven fiber composites with a regular arrangement of reinforcing fibers (Peters, 1998). In all cases, all fillers encountered in given composite have the same material properties and the matrix is nominally homogeneous. Hence, the parameters defining the composite design space are the filler size, shape and spatial distribution.

Biological materials are much more complex as they contain multiple ‘constituent phases’, exhibit graded compositions and are structured on multiple scales. In addition, living materials accommodate stochasticity in structure and constituent properties to a level that makes one think that such fluctuations are essential for the proper functioning of these materials. Most connective tissues are made from collagen and elastin fibers forming complex networks. The composition and structure of these networks are highly stochastic in cartilage, for example, and

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almost regular in arterial walls and the human intervertebral disk (Humzah and Soames, 1988). Bone is a composite exquisitely structured on scales from the nanometer to the macroscopic scale, in which fluctuations of density and composition play a central role in providing the desirable combination of strength and toughness (Launey et al., 2010). It is a stochastic porous material with a solid phase currently viewed as a bi-continuous composite of interpenetrating hard mineral and soft collagen phases (Chen et al., 2011; Hamed et al., 2012).

Traditional manufacturing technologies mandate the use of a small number of constituent materials in man-made composites. Furthermore, design considerations generally indicate that variability is detrimental as may lead to premature failure. This paradigm can be now brought into question by the new possibilities offered by additive manufacturing. These techniques allow producing materials with virtually any composition and spatial distribution of constituents. Hence, they broaden the design space of composite materials to a level that could not be foreseen a decade ago. Therefore, it is timely to explore this design space, with the hope that new families of composites with enhanced properties can be identified.

The goal of the present study is to explore part of the design space associated with the composite composition. To this end, the constraint on the number of constituents in the composite is relaxed and we study the effect on the effective behavior of allowing for spatial fluctuations of material properties, while keeping the mean of the distributions of these properties constant. This is a follow-up of the investigation presented in Picu et al. (2014), where the effect of filler spatial distribution was investigated. It was seen that spatial correlations of filler positions lead to stiffer composites with higher strain hardening rates and with higher effective damping ratios. The largest effect was obtained for power law correlations, case in which the fillers have a fractal distribution.

To address the goal stated above, we consider three types of model composites: Type 1 – composites with homogeneous elastic–plastic matrix and linear elastic inclusions of stiffness sampled from a distribution function. The mean of this distribution is kept constant and we investigate the effect of its variance on the homogenized response of the composite; Type 2 – composites with spatial variability in the elastic–plastic material parameters of the matrix. Similarly, the mean of the distribution of these parameters is kept constant and the variable of the problem is its variance. Type 2(a) contains no inclusions, while Type 2(b) composites have inclusions. In the Type 2(b) case fillers are much smaller than the wavelength of matrix fluctuations and their stiffness is significantly higher than the mean stiffness of the matrix in all cases. All inclusions in given model have the same stiffness. In all models considered, the spatial distribution of inclusions and of matrix property fluctuations is random. This allows separating the effect of variability in the constituent properties from that of the spatial correlations discussed in Picu et al. (2014).

Homogenization of composite materials has a long history. Reviews on this topic are presented in Nemat-Nasser and Hori (1999), Torquato (2002) and Dvorak (2013). Remarkable results have been obtained regarding the bounds on the elastic moduli of such composites. These expressions are generally given in terms of the volume fraction of the constituents. The closest bounds for the bulk modulus which take into account only the volume fraction have been derived by Hashin and Shtrikman (HS) (Hashin and Shtrikman, 1962). A family of higher order bounds, which take into account statistical measures of the microstructure geometry, have been proposed more recently with the purpose of reducing the separation between the upper and lower bounds (e.g. Beran and Molyeux, 1966; Silnutzer, 1972; Milton, 1981, 1982; Phan-Tien and Milton, 1982; Quinatanilla and Torquato, 1995). The *n*-point bounds are written in terms of *n*-point microstructural correlation functions which define the probability that *n* points with specified relative positions are all located in a certain phase of the composite. Any statistical correlation of the microstructure can be accounted for by using these methods. A review of the higher order bounds and the geometric parameters required for their evaluation is provided in Torquato (2002).

The effective behavior of heterogeneous materials with linear elastic constituents and random microstructure has been studied in a number of works using continuum and discrete (lattice) models (Ostoja-Starzewski, 1994; Huyse and Maes, 2001; Dimas et al., 2014, 2015). It was reported that the homogenized moduli of the composite depend weakly on the type and degree of variability in local properties and stronger on the way the random field defining the fluctuations is imposed. As the variability of the random field defining the local material properties increases, the effective linear elastic moduli of the composite decrease (Huyse and Maes, 2001). The variance of the components of the stiffness tensor is similar and decreases with increasing the sample size (Dimas et al, 2015), as suggested by the ergodic hypothesis.

In this work we use numerical homogenization of representative volumes of the random composites described above and focus on the effective elastic–plastic behavior and the distribution of maximum principal stress, which is relevant for damage initiation.

2. Models and methods

Two dimensional composites are considered in this study. The three types of models analyzed are shown in Fig. 1. Fig. 1(a) (Type 1) shows a particulate composite with randomly distributed inclusions of volume fraction f. The level of gray in the figure indicates Young's modulus of the various phases. The matrix is homogeneous and elastic–plastic with a bilinear constitutive equation characterized by moduli $E_e = E_1$ and $E_p = 10^{-2}E_1$ for the elastic and plastic branches of the constitutive law (except where stated otherwise), respectively, and by the yield stress, σ_{y1} ($\sigma_{y1} = 10^{-2}E_1$). Inclusions are linear elastic, of modulus E_2 sampled from a log-normal distribution function, $p(E_2)$. The distribution is not truncated at large values of the modulus. The mean of this distribution, $\bar{E}_2$, is kept constant at $\bar{E}_2 = 10E_1$, while its variance is a parameter. The second moment is represented by the coefficient of variation c_E of $p(E_2)$. All components have the same Poisson ratio, $\nu = 0.3$, and plane strain conditions are considered throughout.