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Overall properties of particulate composites with fractal distribution of fibers

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ABSTRACT

Extensive research on the micromechanical structure of materials has revealed that some of the main construction materials exhibit fractal patterns at the micro scale. Therefore, micromechanical structure of these types of materials can be viewed as periodic structures at different length scales. A new homogenization technique is proposed that is mainly based on the micromechanical averaging schemes for the determination of the mechanical properties of materials with periodic microstructures. This method can be used in determining the overall properties of particulate composites. The proposed technique is a multi-step homogenization technique in which in each step a length scale is considered until the whole reinforcing phase is taken into account. To validate the method, its results are compared with the experimental data on different composite materials with different matrices and fibers. The results show that very good estimates of the mechanical properties is reached by utilizing the proposed multi-step homogenization technique. This method can be utilized in the determination of the mechanical properties of the composites with the coated fibers, where the effect of size of particles on the mechanical properties can be investigated by the application of principles of fractal geometry.

1. Introduction

The interest in mixing materials to obtain new materials with desired mechanical properties is as old as human history. However, it has only been a few decades that a systematic approach for the determination of the properties of such materials is available. Starting from the seminal works of Eshelby (1957, 1959, 1961) on ellipsoidal inclusions many works have been done on various aspects of micromechanical analysis of inhomogeneous solids. One of the active research subjects in the field of micromechanics of solids is about determining the overall properties of composite materials. The early works on this subject goes back to the Voigt and Reuss bounds on elastic moduli that result directly from the rule of mixtures method. More delicate works on the subject were done later, among which one can mention the variational universal bounds of Hashin and Shtrikman (1961, 1962a, 1962b, 1963). Other significant contributions in the field are Hill (1963); Willis (1977); Talbot and Willis (1985, 1987); Weng (1990); Castañeda (1991, 1992). Another direction in the field of micromechanics of composite materials was started by the introduction of the equivalent inclusion method by Mura (1964), which was followed and extended extensively in the later works of Nemat-Nasser and Taya (1981); Nemat-Nasser et al. (1982, 1993); Nemat-Nasser and Hori (2013). Following the works of Nemat-Nasser and Taya (1981); Nemat-Nasser et al. (1982) on periodic distribution of fibers, some research studies devoted to extend the scope of periodic homogenization to the cases of non-dilute coated fiber reinforced composites (e.g. see El Mouden et al., 1998; Shodja and Roumi, 2005).

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Since the pioneering work of Mandelbrot (1983), many research studies have been carried out in various applications of fractal geometry. Fractal geometry has found many applications in various fields of science that deal with objects of irregular shapes. Introduction of this mathematical concept has extended the scope of research to objects of irregular shapes and geometries, which in the past were assumed to be mathematically intractable. The interest in the application of fractal geometry in mechanics started from the realization of the fact that the fracture surfaces of some materials exhibit fractal patterns (Saouma and Barton, 1994; Saouma et al., 1990). There are extensive works in the subject of the effect of fractality on the various aspects of fracture mechanics properties of materials (see Mosolov, 1991; Gol'dshtein and Mosolov, 1991, 1992; Balankin, 1997; Borodich, 1997; Carpinteri, 1994; Cherepanov et al., 1995; Xie, 1989, 1995; Yavari et al., 2000, 2002; Yavari, 2002; Wnuk and Yavari, 2003, 2005, 2008, 2009; Yavari and Khezrzadeh, 2010; Khezrzadeh et al., 2011; Balankin, 2015 and references therein.). Now the scope of research on fractal objects is extended to the mechanics of materials with fractal microstructure, and the continuum mechanics framework for the fractal media is being developed (see Tarasov, 2005, 2011; Ostoja-Starzewski et al., 2014).

The aim of this paper is to introduce a new method for characterizing overall behavior of particulate composite materials, especially those with fractal distribution of fibers. In this paper the advantages of both fractal geometry and periodic homogenization techniques are put together to develop a method for the determination of the overall properties of particulate composite materials with particles of different sizes. The presented method can be applied for the determination of size effect on the mechanical properties of particle-reinforced composites.

The paper is organized as follows. In Section 2, the geometrical model is introduced, by utilizing this geometric model, the microstructures with periodicity at different length scales can be modeled. In Section 3, at first the basics of homogenization techniques for periodic microstructures is introduced and then properties of the fundamental unit cell, which is called Unit Cell I is derived. In Section 4 the theory of multi-step homogenization is developed by implementing the results of the previous sections. In Section 5, the results from the presented method is verified against experimental results of different composite materials and it is shown that good estimations of the elastic moduli are obtained by utilizing the proposed homogenizing technique even at high concentration of fibers. Finally some concluding remarks will be given.

2. Geometrical modeling of particulate composites with fractal-like microstructure

Fractal geometry, as a mathematical tool that can describe the objects of irregular shapes has found many applications in science and engineering. In fractal geometry the irregularity is modeled by using the self-similarity and self-affinity properties of the fractal sets. Self-similarity and self-affinity indicate that the irregular object has some degree of order in different length scales. This indicates that the total object is consisted of many objects, which are identical in shape but are different in scale. Fractal objects measure will be different when they are viewed at different length scales, this measure can be obtained by the definition of a length scale (denoted by yardstick ℓ) and a fractal dimension (D_f)[1] which indicates the degree of the deviation of the fractal object from the surrounding Euclidean space (with the dimension D_E). The real objects with fractal structure can be extracted from the mathematical fractals by defining upper and lower cutoffs on the length scales, the resulting objects are known as prefractals (Ostoja-Starzewski, 2007).

To define the geometric model for particulate composite materials, the construction procedure for a fractal object is used. Consider the schematics shown in Fig. 1 (bottom right) and suppose that this pattern replicates in the smaller length scales as shown. To implement the periodic homogenization method the microstructure of material is idealized by a periodic array of Unit Cell N. The procedure for building geometric model of fiber distribution in the microstructure is described in the followings.

Since we are working with physical fractals there exists a lower cutoff for the fiber size (i.e. a_1), which is embedded inside the matrix and forms a unit cell that we call Unit Cell I. By analyzing this unit cell and obtaining its homogenized mechanical properties we can extend our analysis to the next length scale (a_2), i.e. Unit Cell II. The properties of the homogenized results from previous step are served as the matrix properties of the new unit cell, this procedure continues until the last particle (with radius a_n) in the material is considered. It should be noted that although we started our model from the self-similar models there is no limitation on the distribution and size range of the particle and also their properties. This geometric representation of particulate materials has benefits among which one can mention that in this method the homogenization takes place in multi-steps and length scales and so it is possible to consider the properties of different materials at each step. As an example, consider the particles in the concrete, the ITZ is commonly believed to have the constant thickness inside the concrete (see Monteiro et al., 1985; Akçaoğlu et al., 2004; Basheer et al., 2005; Cwirzen and Penttala, 2005; Hussin and Poole, 2011) so the aggregates with different radii will exhibit different mechanical properties in the concrete. This cannot be considered in the well known formulas. However, by using this method it is possible to consider this effect by taking the aggregates of the same radius at each step of this multi-step homogenization method.

This geometric model is an efficient tool for representation of the microstructure of particulate materials with fractal distribution of particles. It should be noted that depending on the volume fraction of particles and also the size distribution of them the model can be changed accordingly. In the following sections the mathematical

[1] There exist different mathematical definitions for fractal dimension, e.g. box dimension (D_B), compass dimension (D_C) and mass dimension (D_M). In the case self-similar fractals all these dimensions will be identical while this is not the case for self-affine fractals, see Mandelbrot (1983); Falconer (2004) for more details.