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A probabilistic constitutive model for closed-cell foams

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ABSTRACT

In the homogenization analysis of closed-cell foams for determination of their effective properties, the generation of numerical models which reproduce the real microstructure is very complex and time extensive. For this reason, the present study deals with the definition of a probabilistic constitutive model which makes it possible to compute the effective stiffness components and the corresponding scatter band widths without modeling the real microstructure. However, probability distributions of the relative density, the cell size, cell shape and orientation, as the most essential microstructural variables influencing the effective stiffness, are required as the input database. These material characteristics can be easily determined by computed tomography processes. Furthermore, the influence of these distributions for the input variables on the effective material properties is investigated by a systematic variation.

1. Introduction

Due to their high void volume fraction, solid foams are interesting materials for various applications. In lightweight construction, their main advantage is their low specific weight. For this purpose, solid foams are common core materials for sandwich structures (e. g. Vinson, 1999) where their reasonable stiffness at rather low specific weight is exploited. Other mechanical applications include packaging and impact protection systems (Mills et al., 2003) exploiting the effect that due to the high void volume fraction high compaction ratios can be obtained and that the compaction occurs at an approximately constant stress level. Other applications of solid foams are lightweight foamed concrete as a common civil engineering material (Mugahed Amran et al., 2015). On the other hand, cellular structures are suitable for thermal as well as noise protection. Hence, solid foams are able to satisfy different specifications simultaneously.

For the design of structures and structural components consisting partially or in total of solid foams, the disordered microstructure and the resulting scatter of the material properties are a challenge, since no methods for accurate prediction of the uncertainty are available. The scatter in the material response of solid foams has been characterized experimentally in a comprehensive manner by Fazekas et al. (2002) as well as by Ramamurty and Paul (2004). In order to save experimental effort in the material characterization and to support the development of such materials, the use of numerical methods has pronounced advantages. Due to the possibility of a numerical prediction of the effective material properties and a systematic variation of the influencing variables, the number of experiments, which are very cost extensive, can be reduced. Nevertheless, appropriate models are required, which adequately account for the microstructural uncertainty.

An early probabilistic model accounting for the uncertainty in the cell wall orientation of two-dimensional cellular structures has been provided by Fortes, Ashby, 1999). Since then, a variety of contributions on the numerical prediction of the effective material response of solid foams based on improved microstructural models has been

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provided in the literature. In this sense, the most important classical foam model is Kelvin's (Thomson, 1887) tetradecahedron which has been derived from the idea to construct a spatially repeating regular cell model with a minimum surface-to-volume ratio. In order to obtain an isotropic material response, Christensen (1987) proposed a pentagonal dodecahedron as a repeating unit cell. Gibson and Ashby (1997) used simplified brick type models in conjunction with a dimensional analysis to derive approximate formulae for the basic mechanical characteristics of open and closed cell foams. In a revisitation of Kelvin's principles, Weaire and Phelan (1994) derived a spatially repeating four-cell model which outperforms Kelvin's classical model in terms of its average surface-to-volume ratio. On the other hand, the use of numerical models which directly reproduce the real microstructure of given foams using computed tomography is frequently used in recent contributions (see Montminy et al., 2004;Youssef et al., 2005). The advantage of this method is that it accounts for all microstructural features in a direct manner. Nevertheless, it has the disadvantage that it is restricted to the features of a small partition of the material which is not necessarily representative for the entire microstructure. Furthermore, it is rather time-consuming because not only the generation of such models but also their stress-strain analysis is computationally expensive. Approaches in this sense have been provided e. g. by Bock (2011), Montminy et al. (2004) and Youssef et al. (2005).

A major shortcoming of the mentioned methods for modeling the microstructure of foams is the fact that these models do not account for the geometric uncertainty of the microstructure since all of them are inherently deterministic approaches. A method to circumvent this problem is the generation of the cell structures using the Voronoï process (Voronoï, 1908; Hardenacke and Hohe, 2010; Wejrzanowski et al., 2013). With this space division technique, it is possible to generate representative microstructural models which are based on the probability distributions of the most essential microstructural variables, such as the cell size and the relative density (Beckmann and Hohe, 2012; Fan et al., 2004; Redenbach, 2009). This method has the advantage that the influence of the input quantities on the computed effective material properties can be analyzed systematically.

The Voronoï process is based on the random generation of nuclei. Based thereon, the individual cells are defined as the sets of spatial points which are closer to the respective nucleus than to all other nuclei. In general, the cell shape cannot be controlled by this process in an a priori manner. For this reason it is difficult to include the influence of stretched cells which e. g. can arise due to the manufacturing process. In the present study, a numerical model is proposed which affords to predict the effective stiffness components of a closed-cell foam based on the most essential microstructural variables as the distributions of the relative density, the cell size, the cell aspect ratio and orientation without modeling the real microstructure. A probabilistic constitutive model is derived based on an existing cell-based assessment of finite elements models which has been published previously by the authors (Beckmann, 2015; Beckmann and Hohe, 2012).

2. Motivation and basic procedure

The motivation for definition of a probabilistic constitutive model is given by the results of a previous contribution by the authors (Beckmann and Hohe, 2012) on the determination of the effective material properties of solid foams. In this publication, the material properties are analyzed locally using the individual cells as 'testing volume elements'. In this study, a strong correlation between the effective stiffness components of the cells and the relative density has been found. Furthermore, it has been observed that the effect of the neighborhood of a specific cell on its effective properties is only minor.

In particular, these observations suggest a further investigation of the correlations, considering also the correlation between the cell geometries and the local effective stiffness components as already proposed in the 1980s by Cowin (1985), Turner and Cowin (1987) and Huber and Gibson (1988). In these publications, a relationship between the anisotropy of foams and the stiffness tensor has been derived. In 1995, Zysset and Curnier (1995) have published a further model in order to describe the anisotropy of the elastic constants based on a disordered microstructure. Several years later, Rincón Kohli (2003) has stated in her graduate thesis: 'The mechanical properties of cellular materials depend not only on the properties of the solid it is made of, but also on the amount of this material and the geometrical arrangement of the structure.' This means that the relative density is the most important microstructural property affecting the stiffness, but due to the fact that this is a scalar quantity, she predicts that in addition, the information about the cell geometry is absolutely essential. In order to analyze the influence of non-universal Voronoï cell shapes, a description for generating such models is shown in a recent publication by Schaller et al. (2015).

On account of these observations, a separation of the effects of the relative density, the aspect ratios and the orientations of the cells as the most essential microstructural variables is performed. The approach is based on the microstructures obtained in a cell-based homogenization analysis provided earlier by the present authors (Beckmann and Hohe, 2012). The mentioned microstructural variables, i.e. the relative density, the cell aspect ratio and the spatial orientation of the cells are treated as random variables, provided with appropriate probability distributions. In addition, it is assumed that a residual scatter, which is not included in these variables, persists.

For this purpose, an initially deterministic description is employed, based on the choice of appropriate mathematical relationships to describe the influence of the mentioned basic microstructural variables on the components of the effective stiffness tensor (Section 3). Subsequently, a probabilistic constitutive model is developed in consideration of this separation ansatz by treating the basic microstructural variables, i.e. relative density, cell aspect ratio and spatial orientation, as stochastic variables. In doing so, it is possible to determine the distributions of the stiffness components directly out of the known – or at least experimentally measurable – distributions of the stochastic variables. For structural application, i.e. in the