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The Krajcinovic approach to model size dependent fracture in quasi-brittle solids

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ABSTRACT

The failure in "quasibrittle" microstructural systems, occurring with no early warning, is a debated problem of great practical importance for the structural engineering community. Available models do not fully account for typical sample-size effects observed in fracture initiation and propagation. The Krajcinovic approach (K-approach) proposed here stems from a posthumous interpretation of Krajcinovic's original ideas and offers a new route to tackle such effects by means of an advanced fractal scheme, which consists of the sequential application of the Family–Vicsek scaling laws for the number of damage events $n(\varepsilon; L)$ in the fracture initiation and propagation regimes separately. The procedure is developed and explained in the context of an established lattice models under static tensile testing. Average simulation data for any outer-size L – here ranging from 24 to 192 – is shown to scale nicely by this method, throughout the entire damage process. The proper definition of the damage parameter D allows deploying the deduced scaling laws to deduce the actual stress vs. strain relationship applicable in engineering. The discussion extends with no prejudice to data from real experiments, provided that all necessary information is gathered and all underlying assumptions hold true. The approach shall appeal per se also to the larger scientific community of physicists and mathematicians involved in statistical mechanics and random network failure.

1. Introduction: quasi-brittle failure, size effects and fractals

Brittle, embrittled, and "quasibrittle" microstructural systems have the tendency to fail catastrophically with little or no early warning as they reach their strength, as shown in Fig. 1. Besides the case of a structure containing a critical defect, such failures often develop from diffuse microcracking resulting in fracture initiation and propagation. Modeling and predicting failure of these systems is of utmost importance and has proven to be a formidable task of damage mechanics. In fact, a major complication is represented by the sample-size dependence of both the onset of strain localization and the consequent damage evolution. It is hard to predict the behavior of large structures based on laboratory tests on similarly shaped samples, unless a size-effect model (i.e., a scaling law) can be established to obtain analytical estimates. If a scaling law is

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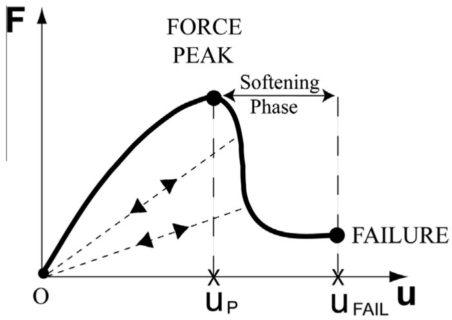

Fig. 1. Typical tensile response of a sample made of quasi-brittle material such as concrete, failing from fracture initiation. The softening phase begins with the force peak point and ends with the structural failure. The signature feature of this behavior is the progressive loss of stiffness, as observable from the unloading paths identified solely by the secant slope in each point with no residual strain.

available, knowledge of the statistics of a process on one scale allows inferring the statistics of the same process on any other scale. Materials systems of interest involve, for example, concrete, composites, rocks and timber, as demonstrated by a massive experimental evidence reported in literature over the past 40 years.

The modeling problem has been under investigation for centuries and many researchers have attempted a number of different strategies. Some modern approaches to fracture and damage have originated from fractal theory and lattice models. For instance, Mishnaevsky Jr. (1996) monitored the surface roughness of crack and the specific surface energy needed to form a crack by the mechanism of microcrack coalescence and concluded that the fractal dimension of crack may be monitored during the crack formation process to compute the time-to-fracture in heterogeneous solids. Another group (Cherepanov et al., 1995; Balankin et al., 1996) suggested that the usual LEFM expressions for stress concentration at the crack tip could be replaced by a fractal version based on a roughness-related power law exponent α and a fractal stress intensity factor K_f as

$$\boldsymbol{\sigma}_{ij} \propto K_f \frac{r^{-\alpha}}{l_0} \tag{1}$$

when crack length l falls between a lower cut-off l_0 and a self-affine correlation length ς, $l_0 < l < \varsigma$. Similar continuum-based approaches have followed (Borodich, 1997) with some noteworthy contributions that include for example the "Quantized Fracture Mechanics for fractal cracks" (Pugno and Ruoff, 2004; Wnuk and Yavari, 2008) or the fractional continuum framework of fracture and damage discussed by Tarasov (2013), Ostoja-Starzewski (2007), and Ostoja-Starzewski et al. (2013).

As far as damage initiation in quasi-brittle materials, Carpinteri and coworkers (e.g., 1994, 2012) devoted substantial effort to size-effects inherent to fracture in concrete and proposed what they called multi-fractal scaling laws for the strength $\boldsymbol{\sigma}_{\mathrm{PEAK}}$, which we can rewrite here as:

$$\boldsymbol{\sigma}_{\mathrm{PEAK}} \propto \sigma^{*}_{\mathrm{PEAK}} L^{-\beta} \tag{2}$$

where $\boldsymbol{\sigma}^{*}_{\mathrm{PEAK}}$ is a scale invariant material parameter, β is the fractal exponent ranging from 0 to ½, for the low and high material heterogeneity limits respectively, and L is the sole significant geometrical parameter, provided that only self-similar samples are compared. When applied to experimental data, the authors stated that Eq. (2) provides an empirical method to obtain reasonable values for the fractal dimension damage domain, barring the existence of excessive scatter in the experimental data. They also documented a good agreement with the microcracking process as measured from the acoustic emission (AE) experiments, recovering the power-law and intermittency of avalanches of AE events, as well as the fractal distributions of event locations.

Nonetheless, the multi-scale approach faced also some criticism from exponents of civil engineering community, primarily by Bazant and co-workers. Following an alternative rationale, Bazant (1997a,b) first developed an asymptotic argument and proposed a different model for the fracture initiation problem

$$\boldsymbol{\sigma}_{\mathrm{PEAK}} = \boldsymbol{\sigma}^{*}_{\mathrm{PEAK}} \left(1 + \frac{A}{B+L}\right)^{-1/\beta} \tag{3}$$

where A and B are fitting constants. In subsequent papers, Bazant (2004)reported a thorough overview of size effects and scaling laws for many different structural systems based on their approach, pointing out the affinity with Weibull statistics and strongly advocating weaknesses of fractal-based models such as (2).

However, a third-party work by Karihaloo and co-workers (Ince et al., 2003) found merits in both approaches when comparing approach (2) vs. (3), one outperforming the other in different size ranges, which ushers in the possibility for such dispute to live on unsettled. At the same time there is the general view in the engineering community that modeling size-effects remains a fertile and urgent research ground for the sake of establishment of reliable models and improvements of current design codes in structural engineering.

In this paper we present a different scaling procedure that we will call "Krajcinovic approach" (K-approach hereafter) as this stems from our posthumous revision of seminal ideas and prior work headed by Dusan Krajcinovic. The original papers by Krajcinovic and Rinaldi (2005a,b) and Rinaldi et al. (2006, 2007) laid the foundation of the work presented here and fostered the usage of fractal theory in connection with lattice model, in a manner very different from Carpinteri's. Rather than concentrating on the fracture strength, Krajcinovic's initial exploration focused on establishing the connection between a random heterogeneous microstructured material and the damage parameter D in the constitutive relation throughout the damage process, from the early stage microcracking to the final crack propagation. Noteworthy, D enters the constitutive relations of a material but it is not an intrinsic property, being associated to given boundary conditions and a given loading history. For example, unlike comminution damage in fragmentation problem where the complete loss of stiffness is not achieved at failure (e.g., Mastilovic and Krajcinovic, 1999a,b), the damage parameter for the tensile test is strictly related to loss of stiffness such as $D = \Delta E/E_0$ (Rinaldi, 2009).