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Large strain compressive response of 2-D periodic representative volume element for random foam microstructures ☆

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ABSTRACT

A numerical investigation has been conducted to determine the influence of Representative Volume Element (RVE) size and degree of irregularity of polymer foam microstructure on its compressive mechanical properties, including stiffness, plateau stress and onset strain of densification. Periodic two-dimensional RVEs have been generated using a Voronoi-based numerical algorithm and compressed. Importantly, self-contact of the foam's internal microstructure has been incorporated through the use of shell elements, allowing simulation of the foam well into the densification stage of compression; strains of up to 80% are applied. Results suggest that the stiffness of the foam RVE is relatively insensitive to RVE size but tends to soften as the degree of irregularity increases. Both the shape of the plateau stress and the onset strain of densification are sensitive to both the RVE size and degree of irregularity. Increasing the RVE size and decreasing the degree of irregularity both tend to result in a decrease of the gradient of the plateau region, while increasing the RVE size and degree of irregularity both tend to decrease the onset strain of densification. Finally, a method of predicting the onset strain of densification to an accuracy of about 10%, while reducing the computational cost by two orders of magnitude is suggested.

1. Introduction

Foams are an important class of engineering material used in a wide range of mechanical applications including lightweight sandwich structures where high specific stiffness and strength are important in the sandwich core, as flexible cushions to distribute pressure loads (Miltz and Ramon, 1990), and as impact energy absorbers where they are often used to limit the transmission of inertial forces (Mills et al. 2003). The microstructure of foam is fundamental in determining its bulk mechanical response. Consequently, a large body of work has been dedicated to understanding the relationship between microstructure and macro-scale properties of these materials. Computational homogenisation provides a powerful tool to investigate this relationship. Importantly, since the technique can include significant geometric changes of the material structure across the length scales, it is applicable to the study of materials undergoing large deformations (Hardenacke and Hohe, 2009). Cellular materials, such as polymer and metal foams, are obvious candidates for analysis using this technique, due to their common use in impact and cushioning applications where large deformations are anticipated and are typically included at the product design stage.

Ideally, micro to macro (Laroussi et al. 2002; Zhou and Soboyejo, 2004; Dillard et al. 2006), or meso to macro

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(Boubakar et al. 2002) simulation strategies can lead to significant time and cost savings typically incurred during experimental characterisation, where virtual testing is intended to replace the majority of the experimental test matrix. The technique can also be applied in material optimisation by informing manufacture processes in order to induce beneficial changes in a material's micro (Duarte and Banhart, 2000; Blazy et al. 2004; Wouterson et al. 2005) or meso-structure (Boisse et al. 2011).

The concept of a representative volume element (RVE) (Hill, 1963) employed in combination with a Periodic Boundary Condition applied along its edges (Guedes and Kikuchi, 1990; Anthoine, 1995) is often used to obtain a homogenised macro-response for a material's bulk behaviour. Here, the volume averaged deformation gradient across the RVE is determined from the displacement of its surface, likewise the volume averaged nominal stress is computed in terms of the nominal stress on its surface. Once these volume averaged behaviours are determined they can be used either in parameter fitting for continuum-based constitutive models, or more directly, using a micro-to-macro simulation strategy (Miehe and Koch, 2002; Hohe and Becker, 2003).

The accuracy and practicality of computational homogenisation depends on the use of RVEs that are both realistic and computationally efficient, two criteria that are often at odds with one another (Kouznetsova et al., 2001; Smit et al. 1998; Swan, 1994). The size of the RVE and its level of detail are two important considerations. For materials based on a regularly repeating micro or mesoscale structure, such as a honeycomb core or a woven textile, the choice of RVE size is usually trivial, and can be taken as the repeat unit cell within the material (Smit, 1998) (though this choice precludes the prediction of deformations with wavelengths longer than the size of the repeat unit cell). When it comes to materials possessing random micro-structures, in general, the larger the RVE the more microscopic structural information it will contain. Ideally, an RVE model should be sufficiently large to be statistically representative of the composite (Drugan and Willis, 1996) while small in comparison to the larger structure. However, restrictions on computational resources impose practical limitations on the size of the RVE and so the model should instead be chosen such that it can predict the overall response within a desired accuracy. To the best of the authors' knowledge, there is no analytical method of predicting the minimum RVE size for cellular structures undergoing large deformations. An important aim of this work is therefore to investigate the effects of changing the RVE size, as a function of the property under investigation (and consequently the level of compression) and the degree of irregularity within the RVE model.

When it comes to the level of detail within the RVE model, one extreme strategy is to employ RVEs based on, for example, actual 3-D topologies, measured using techniques such as micro-CT imaging (Shan and Gokhale, 2001; Maire et al. 2003; Michailidis et al. 2011). In this case, issues associated with strict application of the PBC have to be resolved (Youssef et al. 2005) and long simulation times associated with the large number of continuum elements in the RVE currently restrict practical application for complex structures. The other extreme involves significant simplifications in the material's microstructure, involving various measures such as the use of numerical algorithms to generate analogous microstructures (Zhu et al. 2000; Korner et al. 2002; Roberts and Garboczi, 2002; Kraynik et al. 2003), minimisation of the size of the RVE, reduction in the dimensionality of the problem and use of structural elements, such as beam or shell rather than continuum elements (Chen et al. 1999; Fazekas et al. 2002; Schmidt, 2004). Most researchers strike a compromise between these extremes in order to produce predictions of acceptable accuracy and reasonable speed (Jang et al. 2008, 2010).

A recognised method of generating representative microstructures is through Voronoi tessellation (Voronoï, 1908). This numerical technique provides a fast and effective method to create beam-based finite element microstructures of geometry similar to those of several classes of polymeric and metallic open and closed cell foams (Silva et al. 1995; Van der Burg et al. 1997; Shulmeister et al. 1998; Zhu et al. 2000). The method has been used previously to generate RVE models with a PBC for parametric investigations into the effects of factors such as structural variability, relative density and beam cross-section, on the macro-scale response of the foam (Zhu et al. 2000; Zhu and Windle, 2002; Kraynik et al. 2004; Gong et al. 2005; Kraynik, 2006).

Typically, the mechanical behaviour of foams under compressive strain can be classified into three distinct regions: the linear, plateau and densification regimes (Gibson and Ashby, 1997). All the aforementioned investigations incorporating a PBC have been limited to the linear and plateau regions due to the absence of self-contact within the structure. Others have included self-contact in order to simulate the impact response of foam while including the effect of densification (Zheng et al. 2005; Li et al. 2007; Borovinsek and Ren, 2008; Song et al. 2010). These simulations were conducted without application of a PBC, a necessary omission due to the inertial response induced during high rate impacts. Thus, the latter were effectively simulations of simple macro-scale structures incorporating detail at the micro-scale. To the best of the authors' knowledge, there have been no investigations reported in the literature that consider the large-strain compressive behaviour of either a two or three-dimensional beam-based RVE that incorporate both self-contact and a periodic boundary condition. Here, a method of simultaneously including both of these features in two-dimensional RVEs, using a commercial FE code is demonstrated.

While a two-dimensional representation is a major simplification compared to real foams, the work is nevertheless a first step towards the development of the same strategy in a full three-dimensional model. Further, by first considering the problem in two dimensions, and then later in three dimensions, an intention is to determine which information, if any, translates from the much less computationally intensive two dimensional case to the three dimensional case. For example, transferable conclusions regarding trends in the optimum RVE size in 2-D could