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Determination of strength distribution of quasibrittle structures from mean size effect analysis

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ABSTRACT

This paper presents a new method to indirectly determine the probability distribution function of strength of quasibrittle structures. The method is derived within the framework of the finite weakest link model, which indicates that the size dependence of the mean structural strength can be calculated from the size effect on the probability distribution of the structural strength. By considering the inversion of this model, we show that the cumulative distribution function of structural strength can be explicitly determined from the parameters of the mean size effect curve. The proposed method is verified by a comprehensive set of experiments on asphalt mixture at a low temperature, which involves tests on both the strength histograms and the mean size effect. In the meantime, the agreement between the predicted and experimentally measured strength histograms of asphalt mixture specimens of different sizes further confirms the validity of the finite weakest link model.

1. Introduction

Understanding the probability distribution of structural strength is of paramount importance for the reliability-based design of modern engineering structures, such as buildings, bridges, dams, aircraft, ships, etc. As generally accepted, most engineering structures must be designed to guard against an extremely low failure probability, which is typically less than 10^{-6} (Duckett, 2005; Melchers, 1987; NKB, 1978). Evidently, it is impossible to determine the design strength corresponding to such a low failure probability directly from histogram testing. Therefore, we must resort to some probabilistic models. For perfectly ductile and brittle structures, the probabilistic models of strength distribution are well understood. The cumulative distribution function (cdf) of strength of ductile structures must follow the Gaussian distribution according to the Central Limit Theorem because the peak load of the structure can be calculated from the weighted sum of the strengths of the material elements along the failure surface. By contrast, the failure of perfectly brittle structures is triggered by the failure of one material element whose size is negligible compared to the structure size. According to the infinite weakest link model, the strength cdf of brittle structures must follow the Weibull distribution (Bažant et al., 2009; Bažant and Pang, 2007; Le et al., 2011). Both the Gaussian and Weibull distributions are two-parameter probability distribution functions, where the statistical parameters can be easily obtained by histogram testing involving a relatively small number of specimens. However, this is not the case for structures made of quasibrittle materials.

Quasibrittle materials are brittle heterogenous materials exemplified by concrete, fiber composites, woven composites, ceramics, rocks, etc. The salient feature of quasibrittle structures is that the size of material inhomogenieties is not negligible compared to the structure size. Recent studies focused on a particular class of quasibrittle structures, where the peak load is reached as a macro-crack initiates from one representative volume element (RVE) (Bažant et al., 2009; Bažant and Pang,

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2006, 2007; Le et al., 2011; Le and Bažant, 2011). The failure of such structures is of one of the most dangerous types since there is no precursor of the failure. It has been shown that this class of structures can be statistically modeled as a chain of RVEs (Bažant et al., 2009; Bažant and Pang, 2006, 2007; Le et al., 2011). In contrast to the case of perfectly brittle structures, this chain is not infinite because the size of RVE is not negligible compared to the structure size. Consequently, the strength cdf of each RVE must be known in order to calculate the strength cdf of the entire structure. Based on atomistic fracture mechanics and a statistical multi-scale transition model, recent studies (Bažant et al., 2009; Le et al., 2011) showed that the strength cdf of one RVE can be approximated as a Gaussian distribution with a Weibull distribution grafted at its left tail. The finite weakest link model indicates that the strength cdf of quasibrittle structures generally consists of two segments separated by a grafting point (Bažant et al., 2009; Bažant and Pang, 2007; Le et al., 2011). The upper segment can be calculated as a finite chain of Gaussian elements, whereas the lower segment follows the classical two-parameter Weibull distribution. The location of the grafting point represents the quasibrittleness of the structure. Determining such a type of strength distribution through histogram testing requires a sufficiently large number of identical specimens, which could be very costly for some quasibrittle materials due to high labor costs (e.g. concrete, sandstones, etc.).

Based on the weakest link model, it has been shown that the cdf of structural strength varies with the structure size and geometry, which leads to an intricate size effect on the mean structural strength (Bažant et al., 2009; Bažant and Pang, 2007; Le et al., 2011). Therefore, it is naturally expected that one may obtain the strength cdf from the mean size effect analysis. In a recent paper (Le and Bažant, 2012), a mathematical model was briefly outlined to relate the strength cdf and the mean size effect curve for quasibrittle structures under uniaxial tension. So far, no experiments have been performed to verify such a model. In this paper, we present a refinement of this model, which can be applied to quasibrittle structures with a non-uniform stress field. The model is then experimentally verified by a comprehensive set of experiments on the asphalt mixture at a low temperature.

2. Review of finite weakest link model and its implication on mean size effect

Let us limit our attention to a class of structures where the failure of the structure under controlled load occurs once a macro-crack initiates from one RVE. In other words, here the RVE is defined as the smallest material volume whose failure triggers the failure of the entire structure. This is different from the classical definition of RVE by the homogenization theory (Hill, 1963). Recent discrete element simulation showed that the auto-correlation length of the random field of the structural strength is approximately equal to the RVE size (Grassl and Bažant, 2009). Therefore, we could consider the RVE strength to be an independent random variable. According to the joint probability theorem, the failure probability of the structure P_f can be calculated as:

$$1 - P_f(\sigma_N) = \prod_{i=1}^{N} [1 - P_1(\sigma_N s_i)] \tag{1}$$

where P_1 = failure probability of one RVE, N = number of RVEs in the structure, σ_N = nominal strength = $cP_{\max}/bD$ for 2D problems, $P_{\max}$ = maximum load that the structure can sustain, D = characteristic size of the structure, b = dimension in the third (transverse) direction, $c =$ constant chosen such that σ_N represents the maximum elastic principal stress in the structure, $s_i =$ dimensionless stress field such that $\sigma_N s_i$ is equal to the maximum elastic principal stress at the center of ith RVE.

Recent studies showed that the failure probability of one RVE can be derived from atomistic fracture mechanics and a statistical multi-scale transition model, where the failure probability of a nanoscale structure can be obtained by applying the transition rate theory to the discrete jump of a nano-crack and the transition between the nano- and macro-scales can be represented by a hierarchy of statistical chains and bundles (Bažant et al., 2009; Le et al., 2011). Based on this model, it has been found that the strength cdf of one RVE can be approximated to consist of two parts: (1) a Weibull tail extending to a failure probability about 10^{-3} to 10^{-4}, and (2) a Gaussian core covering the rest part of the cdf. Mathematically, it can be expressed as (Bažant et al., 2009; Bažant and Pang, 2007; Le et al., 2011):

$$P_1(\sigma) = 1 - e^{-\langle \sigma/s_0 \rangle^m} \approx \langle \sigma/s_0 \rangle^m \quad (\sigma \leqslant \sigma_{gr}) \tag{2a}$$

$$P_1(\sigma) = P_{gr} + \frac{r_f}{\delta_G \sqrt{2\pi}} \int_{\sigma_{gr}}^{\sigma} e^{-(\sigma'-\mu_G)^2/2\delta_G^2} \mathrm{d}\sigma \quad (\sigma > \sigma_{gr}) \tag{2b}$$

where σ = maximum elastic principal stress at the center of the RVE, m = Weibull modulus, s_0 = scale parameter of the Weibull tail, $\langle x \rangle = \max(x, 0)$, μ_G and δ_G are the mean and standard deviation of the Gaussian core if considered extended to $-\infty$; r_f is a scaling parameter required to normalize the grafted cdf such that $P_1(\infty) = 1$, P_{gr} = grafting probability = $1 - \exp[-(\sigma_{gr}/s_0)^m] \approx (\sigma_{gr}/s_0)^m$, and $\sigma_{gr} =$ grafting stress. Finally, continuity of the probability density function (pdf) at the grafting point requires that $(\mathrm{d}P_1/\mathrm{d}\sigma)|_{\sigma_{gr}^+} = (\mathrm{d}P_1/\mathrm{d}\sigma)|_{\sigma_{gr}^-}$. Although there are six statistical parameters in total that define the strength cdf of one RVE (i.e. $m, s_0, \mu_G, \delta_G, r_f$ and σ_{gr}), due to the normalization requirement and the continuity of pdf at σ_{gr}, only four of them are actually independent.

Substituting Eqs. (2a) and (2b) into Eq. (1), we can calculate the strength distribution of the entire structure. It is clear that due to the grafted probability distribution of RVE strength, the resulting cdf of the structural strength would also consist of two parts. Below the grafting stress, the strength cdf follows the Weibull distribution, whereas the strength cdf above the grafting stress follows a chain of Gaussian elements. As the structure size increases, the Weibull portion of the cdf would increase and eventually occupy the entire cdf.

Knowing the strength cdf of the entire structure, we can calculate the mean structural strength $\bar{\sigma}_N$:

$$\bar{\sigma}_N = \int_0^1 \sigma_N \mathrm{d}P_f = \int_0^\infty [1 - P_f(\sigma_N)] \mathrm{d}\sigma_N \tag{3}$$